Asymptotic analysis of collision-induced timing shifts in return-to-zero quasi-linear systems with predispersion and postdispersion compensation

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An asymptotic method for calculating the collision-induced frequency and timing shifts for quasi-linear pulses in return-to-zero, wavelength-division multiplexed systems with predispersion and postdispersion compensation is developed. Predictions of the asymptotic theory agree well with quadrature and direct numerical simulations. Using this theory, computational savings of many orders of magnitude can be realized over direct numerical simulations. © 2006 Optical Society of America

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The primary source of nonlinear impairments in return-to-zero (RZ), wavelength-division multiplexed (WDM) systems using dispersion management (DM) is that of collision-induced timing shift (CITS) due to cross-phase modulation.[1] When DM is employed, pulses undergo a rapid zigzag motion, interacting many times in a series of minicollisions over a large distance. Because of this extended interaction length, pulses may experience either complete or incomplete collisions. This complex dynamics makes analysis of CITS in DM systems difficult.

Many studies[1–3] have been devoted to the analysis of CITS. The methods used in these studies range from direct numerical simulations (DNSs) of the underlying nonlinear Schrödinger (NLS) equation and semi-analytic methods to scaling arguments and asymptotic analysis. Even semi-analytic methods can be computationally expensive when a large parameter space must be explored.

In this Letter, we extend the asymptotic method developed in Refs. 4 and 5 to include quasi-linear (QL) systems with predispersion and postdispersion compensation (PPDC) and make substantial improvements in the accuracy of the method. Using this approach, we can explain the underlying basis for the shape of the time shift function found earlier by several of us.[6]

We start with the perturbed NLS (PNLS) equation, $iu_z+[D(z)/2]u_{tt}+g(z)|u|^2u=0$, written in dimensionless variables, where $t=t_{\mathrm{ret}}/t_*$, $z=z_{\mathrm{lab}}/z_*$, $u=\mathcal{E}/\sqrt{g(z)P_*}$, and $D=-k''/k''_*$; normalization parameters are denoted by an asterisk. $\mathcal{E}$ is the slowly varying complex optical field envelope; t_{ret} and z_{lab} are, respectively, the retarded time and propagation distance. $D(z)$ is the local group velocity dispersion, and $g(z)$ describes periodic lumped amplification and continuous loss.

A single dispersion map consists of two fiber sections (with dispersion coefficients and lengths $D_+>0,z_+$ and $D_-<0,z_-$) and an erbium-doped fiber amplifier (EDFA) located at the beginning of the first fiber. The map length is $z_a=z_++z_-$, and the fraction of the map consisting of the anomalous fiber is $\theta=z_+/z_a$. The path-average dispersion is then defined as $\langle D\rangle=\theta D_++(1-\theta)D_-$, and a measure of the strength of dispersion management is given by[7] $s=[(D_+-\langle D\rangle)z_+-(D_--\langle D\rangle)z_-]/4$. For EDFAs, $g(z)=g_0\exp[-2\Gamma(z-nz_a)], nz_a\leq z<(n+1)z_a$, where n is the map number, Γ is the dimensionless loss coefficient, and $g_0=2\Gamma z_a/[1-\exp(-2\Gamma z_a)]$. The transmission span then consists of the periodic extension of the above dispersion map. PPDC is modeled by appending anomalous fibers of lengths ΔL_{pre} and ΔL_{post} to the beginning and end of the transmission span, respectively.

We first decompose the field envelope into two pulses, one in frequency channel $+\Omega_0$ and the other in frequency channel $-\Omega_0$: $u=u_++u_-$, i.e., u is decomposed into right- and left-going pulses relative to the retarded frame and four-wave mixing effects are neglected. We remark that numerical simulations[3] indicate that two-pulse interactions dominate timing jitter calculations. The standard definitions of average pulse frequency and average pulse time, written for pulse u_+ are, respectively, $\langle\Omega\rangle=-i\int u_+^*(\partial u_+/\partial t)\mathrm{d}t/E$ and $\langle t\rangle=\int t|u_+|^2\mathrm{d}t/E$, with energy $E=\int|u_+|^2\mathrm{d}t$. Using these definitions along with the PNLS equation, it follows that $\partial\langle\Omega\rangle/\partial z=2g(z)/E\int_{-\infty}^{\infty}|u_+|^2(\partial|u_-|^2/\partial t)\mathrm{d}t$, $\partial\langle t\rangle/\partial z=D(z)\langle\Omega\rangle(z)$. The frequency and timing shifts are defined as $\Delta\Omega(z)=\langle\Omega\rangle(z)-\Omega_0$ and $\Delta t(z)=\langle t\rangle(z)-\widetilde{D}(z)\Omega_0+t_0$, where $\widetilde{D}(z)\equiv\int_{-\Delta L_{\mathrm{pre}}}^{z}D(x)\mathrm{d}x$ is the path-integrated dispersion and $t_0=\langle t\rangle(0)$.

To model QL pulse propagation, we take as an initial condition the Gaussian ansatz $u_\pm(z=-\Delta L_{\mathrm{pre}},t)=(\alpha/\sqrt{2\pi\beta})\exp[-(t\mp t_0)^2/2\beta\pm i\Omega_0 t]$. Because of the constant dispersion in the precompensation fiber,

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pulses can undergo at most one minicollision in $\Delta L_{\rm pre}$. Thus, any frequency shift in $\Delta L_{\rm pre}$ can be neglected. Ignoring this small frequency shift from the precompensation fiber, the pulses are linearly evolved over the distance $\Delta L_{\rm pre}$, yielding a chirped Gaussian at $z=0$: $u_{\pm}(0,t)=\alpha/\sqrt{2\pi(\beta+i\tilde{D})}\times\exp[-(t\mp t_0)^2/2(\beta+iD_+\Delta L_{\rm pre})\pm i\Omega_0 t]$. We remark that precompensation reduces pulse peak power, thereby mitigating nonlinear effects. Assuming that $\langle\Omega\rangle$ changes slowly with z, the QL evolution of the pulses from $z=0$ is then approximately given by[8]

$$u_{\pm}(z,t)=\alpha/\sqrt{2\pi(\beta+i\tilde{D})}\times\exp[-(t\mp t_0\pm\langle\Omega\rangle\tilde{D})^2/2(\beta+i\tilde{D})\pm i\langle\Omega\rangle t-(i/2)\langle\Omega\rangle^2\tilde{D}]. \quad (2)$$

Replacing $\langle\Omega\rangle$ with Ω_0 on the RHS of Eq. (2) and substituting into the evolution equations for $\langle t\rangle$ and $\langle\Omega\rangle$ we find, after integrating over t and z,

$$\Delta\Omega(L+\Delta L_{\rm post})=A\Omega_0\int_0^{L+\Delta L_{\rm post}}\frac{g\tilde{D}_0\exp\left[\dfrac{-2\beta\Omega_0^2\tilde{D}_0^2}{(\beta^2+\tilde{D}^2)}\right]}{(\beta^2+\tilde{D}^2)^{3/2}}\mathrm{d}z, \quad (3a)$$

$$\Delta t(L+\Delta L_{\rm post})=\tilde{D}_0(L+\Delta L_{\rm post})\Delta\Omega(L+\Delta L_{\rm post})-\Delta t_{\rm res}, \quad (3b)$$

where $A=4E\beta^{3/2}/\sqrt{2\pi}$ and $\tilde{D}_0(z)=\tilde{D}(z)-\langle D\rangle z_0$. $\tilde{D}_0$ naturally accounts for unperturbed mean pulse motion, with the mean collision location, or the center of the macrocollision, defined as $z_0=t_0/(\langle D\rangle\Omega_0)$. In Eq. (3b), the contribution of the PPDC fiber to $\tilde{D}_0, D_+(\Delta L_{\rm pre}+\Delta L_{\rm post})$, is important and tends to offset the accumulated dispersion from the nonzero (negative) average dispersion. The residual timing shift $\Delta t_{\rm res}$ is defined as[4]

$$\Delta t_{\rm res}(L+\Delta L_{\rm post})=A\Omega_0\int_0^{L+\Delta L_{\rm post}}\frac{g\tilde{D}_0^2\exp\left[\dfrac{-2\beta\Omega_0^2\tilde{D}_0^2}{(\beta^2+\tilde{D}^2)}\right]}{(\beta^2+\tilde{D}^2)^{3/2}}\mathrm{d}z. \quad (3c)$$

To include the nonlinear frequency shift from the precompensation length, the only modification to Eqs. (3) would be to set the lower limit of integration to $z=-\Delta L_{\rm pre}$. The frequency shift from the postcompensation fiber is also small, but we include this frequency shift to illustrate the flexibility of the method. Equations (3) are the key formulas for frequency and timing shifts between a pair of pulses in different channels of a QL RZ system. They provide estimates of frequency and timing shift without the need for detailed numerical calculations and are amenable to asymptotic analysis, which we describe next.

The Laplace method[9] applies to integrals of the form $I(\lambda)=\int_a^b F(z)\exp[-\lambda\phi(z)]\mathrm{d}z$, where $\lambda\gg 1$, $\phi\geq 0$ and $-\infty\leq a<b\leq\infty$. When $\lambda\gg 1$, the main contribution to the integral comes from the neighborhood of a critical point, i.e., where ϕ has a minimum. F and ϕ are then expanded about this point, with typically only the first few terms retained. The Laplace method, with important modifications mentioned below, is employed.

We describe the asymptotic analysis of only Eq. (3a); the analysis of Eq. (3c) is similar. Equation (3a) can be written as $\Delta\Omega(L+\Delta L_{\rm post})=A\Omega_0(\int_0^L+\int_L^{L+\Delta L_{\rm post}})F\exp(-\lambda\phi)\mathrm{d}z$, where $F=g\tilde{D}_0B^{-3}$, $B^2=\beta^2+\tilde{D}^2$, $\phi=\tilde{D}_0^2B^{-2}$ and the asymptotic parameter $\lambda=2\beta\Omega_0^2$. We first focus on the integral from $z=0$ to $z=L$, which can be written as a sum over each dispersion map: $\int_0^L F\exp(-\lambda\phi)\mathrm{d}z=\sum_{n=0}^{N-1}\int_{nz_a}^{(n+\theta)z_a}F\exp(-\lambda\phi)\mathrm{d}z+\int_{(n+\theta)z_a}^{(n+1)z_a}F\exp(-\lambda\phi)\mathrm{d}z$, where N is the number of dispersion maps in the length L. It suffices to consider only the first integral in the sum; the second integral is treated similarly.

We find the critical point z_c of the integral $\Delta\Omega_n^+=A\Omega_0\int_{nz_a}^{(n+\theta)z_a}F\exp(-\lambda\phi)\mathrm{d}z$ by solving $\phi'(z_c)=0$. The relevant solution is $\tilde{D}_0(z_c)=0$, corresponding to so-called minicollision locations where pulse centers coincide. F and ϕ are now expanded, to second and third order, respectively, about $z=z_c$. One then finds after integration

$$\Delta\Omega_n^+\sim\frac{AF_n'J_{1,n}}{\beta\phi_n''\Omega_0}+\frac{3AF_n''\phi_n''J_{2,n}-2AF_n'\phi_n'''J_{4,n}}{6\beta^{3/2}(\phi_n'')^{5/2}\Omega_0^2},$$

with $F_n'=F'(z_c)$, etc., and $J_{k,n}=\int_{x_n^-}^{x_n^+}x^k e^{-x^2}\mathrm{d}x$. In contrast to the standard Laplace method, here the limits of integration $x_n^{\pm}$ are kept finite. The resulting integral $J_{k,n}$ can be expressed as a sum of error functions and exponentials. For completeness, we record the contribution to Eq. (3c) from a single critical point as $\Delta t_{{\rm res},n}^+\sim AF_n'''J_{2,n}/[2(\beta\phi_n'')^{3/2}\Omega_0^2]$, where $F=g\tilde{D}_0^2B^{-3}$.

Because of finite pulse width, pulses will sometimes contribute to Eqs. (3a) and (3c) even if their centers do not coincide; i.e., if $\tilde{D}_0(z_c)$ is small enough, near or pseudocollisions contribute to timing and frequency shifts as well. Said differently, there are dispersion maps where $\tilde{D}_0(z_c)=0$ has no solution, but due to the smallness of $\tilde{D}_0(z)$ there is still a significant contribution to the integrals from this range of integration. If at the integration limits $\phi'\gtrsim O(1/\lambda)$, then the integrals over these dispersion maps can be estimated using integration by parts, with a typical contribution being $AF/2\beta\phi\exp(-2\beta\Omega_0^2\phi)/\Omega_0|_{nz_a}$; we neglect the exponentially smaller term from the other limit of integration. If $\phi'\lesssim O(1/\lambda)$, integration by parts breaks down and one must use a Laplace-like calculation. However, a good approximation in this case is to set $\phi'=0$ and use the standard Laplace contribution with z_c as the integration limit where ϕ is minimum. In practice, we find that including map contributions where $\phi \lesssim O(1/\sqrt{\lambda})$ gives good results; additional map contributions can be included, but their contribution quickly becomes negligible.

The integral over the postdispersion compensation fiber is treated in the same way as the integral $\Delta\Omega_n^+$ above, except that now in F we take $g=1$ and the limits of integration are L to $L+\Delta L_{\text{post}}$.

Summing over all critical points, it follows that

$$\Delta\Omega(L+\Delta L_{\text{post}}) \sim \frac{A}{\beta\Omega_0}\sum_{n=1}^{N_c}\left[\frac{F_n' J_{1,n}}{\phi_n''} + \frac{F_n'' \phi_n'' J_{2,n}}{2\beta^{1/2}\Omega_0(\phi_n'')^{5/2}} - \frac{F_n' \phi_n''' J_{4,n}}{3\beta^{1/2}\Omega_0(\phi_n'')^{5/2}}\right] + \Delta\Omega(\text{p.c.}), \quad (4a)$$

$$\Delta t_{\text{res}}(L+\Delta L_{\text{post}}) \sim \frac{A}{2\beta^{3/2}\Omega_0^2}\sum_{n=1}^{N_c}\frac{F_n'' J_{2,n}}{(\phi_n'')^{3/2}} + \Delta t_{\text{res}}(\text{p.c.}), \quad (4b)$$

where N_c is the number of critical points. Contributions from the pseudocollisions are denoted by $\Delta\Omega(\text{p.c.})$ and $\Delta t_{\text{res}}(\text{p.c.})$.

We now compare our asymptotic theory with DNS of the PNLS equation and with the numerical integration of Eqs. (3). The system considered is similar to the one recently analyzed in Ref. 6. The system has normalized line parameters $L=22.5$, $z_a=0.22$, $\theta=0.66$, $\Delta L_{\text{pre}}=0.22$, $\Delta L_{\text{post}}=0.39$, $s=1.55$, $\langle D\rangle=-0.5$, and $\Gamma=5.5$. The pulse parameters are $\alpha=1.78$, $\beta=1.36$, and $\Omega_0=2.67$. With $t_*=17$ ps, $z_*=228$ km and $k''=-1.27$ ps^2/km, this approximately corresponds to a 10 Gbit/s system that is 5000 km long with a channel spacing of 100 GHz and a loss rate of 0.21 dB/km. To obtain dimensional frequency and timing shifts, Eqs. (4a) and (4b) are multiplied by $1/(2\pi t_*)$ and t_*, respectively.

To solve the PNLS equation, we used a standard split-step Fourier method and for the numerical integration of Eqs. (3) we used the trapezoidal rule. For DNS of the PNLS, loss and gain were included in the PPDC fibers. Shown in Fig. 1 is the total timing shift (normalized to the pulse FWHM=33 ps) as a function of the mean collision location z_0 (normalized to the system length L) for two channels, Ω_0 and $2\Omega_0$. Figure 1 shows that Eqs. (3) are accurate, validating the QL ansatz and the adiabatic approximation used to derive Eqs. (3). For the first channel, Ω_0, the asymptotic approximation of Eqs. (3) is a good approximation and is in excellent qualitative agreement with numerical results. As the channel spacing increases, the asymptotic approximations become more accurate, i.e., as $\lambda\to\infty$ the difference between the RHS of Eqs. (4) and the corresponding integral in Eqs. (3) goes to zero; this is clearly seen in Fig. 1.

A more detailed analysis shows that the two components of Δt, $\Delta\Omega$ and Δt_{res}, have a universal structure. Use of predispersion compensation has the primary effect of translating the main lobes of $\Delta\Omega$ and Δt_{res}; the maximum/minimum values do not change significantly. Clearly, postdispersion compensation

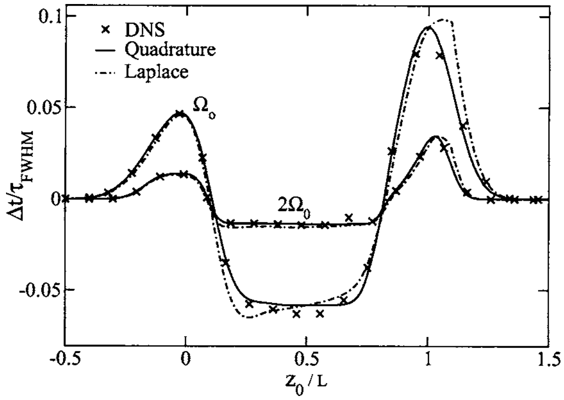

Fig. 1. Total timing shift versus mean collision location z_0 obtained from DNSs of the PNLS equation, numerical integration of Eqs. (3), and asymptotic approximation of Eqs. (3), i.e., using Eqs. (4). Shown are two channels: Ω_0 and $2\Omega_0$.

does not affect frequency and residual timing shifts incurred in the transmission span. Use of PPDC affects Δt primarily by reducing accumulated dispersion, that is, by reducing the timing shift $\tilde{D}_0\Delta\Omega$. Details of these calculations will appear elsewhere.

In summary, we have employed quadrature and asymptotic analysis to calculate CITS for systems with predispersion and postdispersion compensation. For the system chosen here, the quadrature and the asymptotic theory agree well with DNS. Moreover, by using the asymptotic theory, computational savings of many orders of magnitude can be realized over DNS. Because of the dominant two-pulse interactions in WDM systems, the method can also be used to calculate frequency and time shifts for multichannel WDM systems. Higher-bit-rate systems can also be analyzed with the method.

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