Soliton stability conditions in actively mode-locked inhomogeneously broadened lasers

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We examine the conditions for soliton stability in actively mode-locked inhomogeneously broadened lasers. Numerical simulation shows that in the small negative group-velocity dispersion (GVD) region, excessive gain filtering prevents the generation of a single stable soliton pulse and that in the large negative GVD region, the free-running spectral components destabilize the soliton pulse as a result of insufficient gain filtering. As the laser gain medium becomes more inhomogeneously broadened or the unsaturated gain linewidth becomes broader, the gain filtering becomes weaker, and both the lower and the upper stability boundaries shift to small negative GVD. A larger self-phase modulation, however, changes both boundaries toward more-negative GVD and yields a broader stable GVD range. Experimental results for actively mode-locked Nd:silicate glass and Nd:phosphate glass lasers confirm the soliton instability boundary and the influence of the gain medium's degree of inhomogeneity. © 2003 Optical Society of America

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1. INTRODUCTION

Solitonlike pulse formation has been a major technique for generating ultrashort pulses from solid-state lasers for more than a decade and has produced the shortest pulses that have been obtained directly from laser oscillators.[1,2] As a result of the success of soliton pulse shaping in ultrafast lasers, soliton stability has been studied extensively in homogeneously broadened lasers.[3–5] In a homogeneously broadened laser the broadband soliton pulse suffers more gain-filtering loss than does the narrowband continuum, leading to the onset of soliton instability.[3,4] Thus some amplitude modulation, either active or passive, must be present to suppress the growth of the continuum and to keep the solitonlike pulse stable. In an actively mode-locked, homogeneously broadened laser, the group velocity dispersion (GVD) must be more negative than a minimum amount if it is to produce a stable soliton pulse.[5]

A soliton pulse can also be formed in an actively mode-locked inhomogeneously broadened laser.[6] Compared with the homogeneously broadened laser, the actively mode-locked inhomogeneously broadened laser exhibits the unique behavior that a single, stable, solitonlike pulse can be generated only when the GVD is within a certain range of negative values. Outside this stable GVD region, the laser produces mostly incoherent pulses. In this paper we examine the conditions of soliton stability in actively mode-locked inhomogeneously broadened lasers. We discuss the physical origins of the two regions of soliton instability. Using numerical simulations, we study the influence of the gain medium's degree of inhomogeneity, gain linewidth, and self-phase modulation (SPM) strength. Experimently we compared the mode-locking performance of Nd:silicate glass and Nd:phosphate glass lasers, which have different degrees of inhomogeneity. The lower GVD boundary of soliton stability in both lasers as well as the effect of the gain medium's inhomogeneity were confirmed.

In Section 2 we describe the numerical study. The experimental results are presented in Section 3. We give our conclusions in Section 4.

2. NUMERICAL STUDY

We followed the numerical model used in the research reported in Ref. 6, in which the dominant coupled ordinary differential equations were

$$T_R \frac{\mathrm{d}\tilde{E}_n}{\mathrm{d}T} = -\frac{\delta_c}{2}\tilde{E}_n + \frac{\gamma(\nu_n)p_g}{2}\tilde{E}_n - j\delta\phi_n\tilde{E}_n + \frac{\Delta_m}{2}(\tilde{E}_{n+1} - 2\tilde{E}_n + \tilde{E}_{n-1}), \tag{1}$$

$$\Delta\tilde{E}(T, t) = -j\kappa_{\mathrm{SPM}}|\tilde{E}(T, t)|^2\tilde{E}(T, t), \tag{2}$$

$$\gamma(\nu_n) = \sum_{\nu_\xi}\gamma(\nu_\xi, \nu_n) = \sum_{\nu_\xi}N(\nu_\xi)\sigma(\nu_\xi, \nu_n). \tag{3}$$

Time t is the retarded time, and T is the round-trip evolution time. The parameters δ_c, T_R, and p_g are the round-trip power loss, transit time, and gain medium length, respectively, and Δ_m is the modulation index of a sinusoidal active amplitude modulation function. The quantity $\delta\phi_n$ is the round-trip phase shift of the nth mode caused by the GVD. Inhomogeneous gain $\gamma(\nu_n)$ is contributed by various homogeneous group of atoms ξ. Population inversion density $N(\nu_\xi)$ has the Gaussian distribution with an inhomogeneous linewidth $\Delta\nu_{\mathrm{ih}}$. Homogeneous stimulated-emission cross section $\sigma(\nu_\xi, \nu_n)$ has a Lorentzian line shape and a homogeneous linewidth, $\Delta\nu_{\mathrm{h}}$.

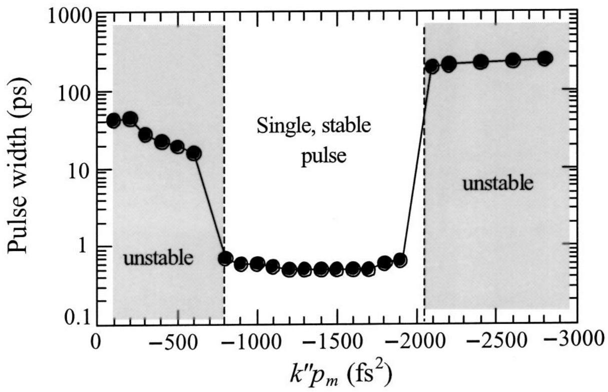

Fig. 1. Simulated dependence of pulse width on negative GVD. Soliton pulses are unstable within the shaded regions.

In the simulation, the peak unsaturated gain is six times above the threshold, resulting in an intracavity average intensity of $I_{\mathrm{ave}} \approx 9 \times 10^4\,\mathrm{W/cm^2}$. The nonlinear coefficient κ_{SPM} is equal to $1.88 \times 10^{-5}\,\mathrm{cm^2/MW}$. Active mode-locking strength $\Delta_m f_m{}^2$ is equal to $2 \times 10^{-3}\,\mathrm{GHz}^2$, where f_m is the modulation frequency. We set the homogeneous linewidth $\Delta\nu_{\mathrm{h}} = 360\,\mathrm{GHz}$ and the inhomogeneous linewidth $\Delta\nu_{\mathrm{ih}} = 1440\,\mathrm{GHz}$, corresponding to a degree of inhomogeneity $\Delta\nu_{\mathrm{ih}}/\Delta\nu_{\mathrm{h}} = 4$. The effective unsaturated gain linewidth is ~1500 GHz, and the free-running lasing bandwidth is ~1250 GHz. These parameters are comparable to those in an actively mode-locked Nd:glass laser.[6] Figure 1 summarizes the simulated soliton mode-locking performance, which shows that instability occurs in both small and large negative GVD regions.

A. Physical Origins of Soliton Instability

The physical origins of soliton instability in an inhomogeneously broadened laser can be traced to the soliton pulse's bandwidth as a function of the GVD, which is given by[7]

$$\Delta\nu_{\mathrm{soliton}} = 0.09\frac{\kappa_{\mathrm{SPM}}W_p}{|k''p_m|}, \tag{4}$$

where p_m is the round-trip effective length of dispersive media. The intracavity pulse energy fluence $W_p = I_{\mathrm{ave}}T_R$ is equal to $9 \times 10^{-4}\,\mathrm{W\,s/cm^2}$ in our simulation with round-trip time $T_R = 10\,\mathrm{ns}$. The soliton pulse always competes with nonsoliton components for the gain. For a homogeneously broadened laser the nonsoliton components constitute a narrowband continuum that occupies the center of the spectrum. In an inhomogeneously broadened laser the nonsoliton components are distributed over a broad spectral region that is supported by the saturated gain. When the GVD is small, the soliton pulse's bandwidth is broad. Because of the gain filtering effect, the broadband soliton pulse suffers more loss than the relatively narrowband nonsoliton components, resulting in an unstable pulse. This excessive gain-filtering effect causes soliton instability in both homogeneously and inhomogeneously broadened lasers. When the GVD is large, the soliton pulse's bandwidth becomes narrow. If the soliton pulse's bandwidth is substantially narrower than the free-running lasing bandwidth, the nonsoliton free-running spectral components will destabilize the soliton pulse. This soliton instability, which is due to insufficient gain filtering in the large negative GVD region, exists only in an inhomogeneously broadened laser, which has a broad free-running lasing bandwidth. In a homogeneously broadened laser the free-running lasing bandwidth is narrow. The soliton pulse's bandwidth is always broad enough to suppress the free-running spectral components, and thus the soliton pulse is stable.

As an estimate according to Eq. (4), if all intracavity energy were contained in the soliton pulse and the soliton pulse were limited to be only within the gain linewidth (1500 GHz), the lower-stability GVD boundary would be $\sim -1000\,\mathrm{fs}^2$. Likewise, if the soliton pulse's bandwidth could be only larger than the free-running lasing bandwidth (1250 GHz), the upper stability GVD boundary would be $\sim -1210\,\mathrm{fs}^2$. In Fig. 2 we show the soliton pulse's spectra near the stability GVD boundaries ($k''p_m = -1000\,\mathrm{fs}^2$ and $k''p_m = -1800\,\mathrm{fs}^2$). Near the lower stability GVD boundary, because of excessive gain filtering, free-running spectral components appear. They spread in a broadband region and correspond to many low-intensity noise spikes in the time domain. As shown in Fig. 2, the soliton pulse is isolated from these small noise spikes and remains stable. By contrast, in the actively mode-locked, homogeneously broadened laser, the narrowband continuum overlaps the soliton pulse in the time domain, leading to a strong perturbation of the soliton pulse that does not permit coexistence of the soliton pulse and continuum. In the inhomogeneously broadened laser, gain filtering can be alleviated through energy transfer from the soliton pulse to the noise background. However, too much noise background destabilizes the soliton pulse. The trade-off allows the lower stability GVD boundary to extend to $-800\,\mathrm{fs}^2$. At the upper stability GVD boundary the soliton pulse has a narrower bandwidth and experiences almost no loss from gain filtering. Hence, in competition with the soliton pulse, the free-running components near the gain center can be suppressed by the combination of a large amount of GVD and the active amplitude modulation. The free-running components about the edges of the saturated gain, however, face little competition from the soliton pulse and may coexist, as shown in Fig. 2(b). Note that the upper stability GVD boundary is beyond the estimate of $-1210\,\mathrm{fs}^2$. The

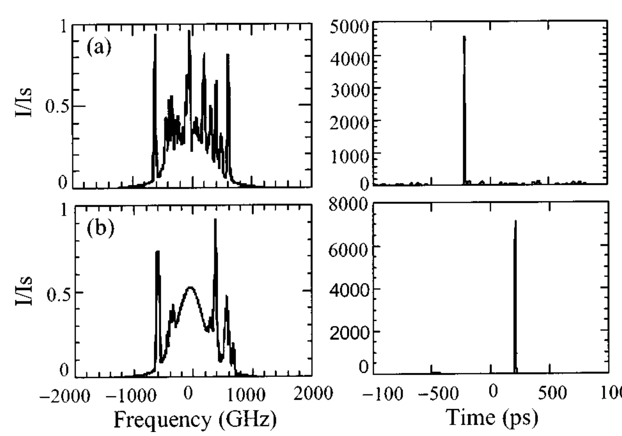

Fig. 2. Soliton pulse spectra when (a) $k''p_m = -1000\,\mathrm{fs}^2$ and (b) $k''p_m = -1800\,\mathrm{fs}^2$.

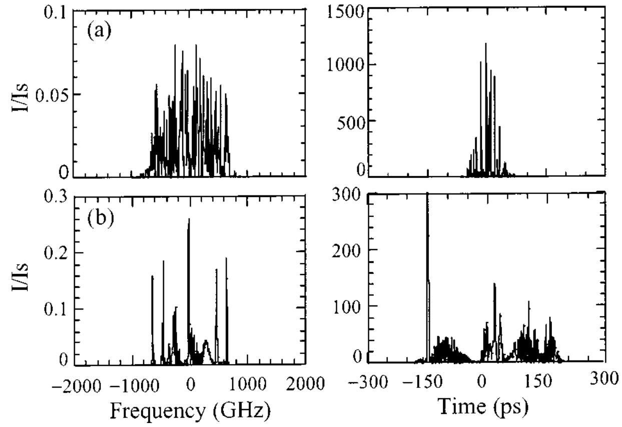

Fig. 3. Pulse spectra and pulse shapes when (a) $k''p_m = -200\,\mathrm{fs}^2$ and (b) $k''p_m = -2400\,\mathrm{fs}^2$.

extension of the upper stability GVD boundary is due to cross saturation of the gain of neighboring atom groups.[8] The soliton pulse can extract energy from neighboring atomic groups within approximately the homogeneous linewidth. Thus the soliton helps to suppress the growth of free-running components, resulting in extension of the upper stability GVD boundary to $-1800\ \mathrm{fs}^2$.

Figure 3 shows typical pulse spectra and pulse shapes of an actively mode-locked inhomogeneously broadened laser outside the stable GVD range. With $k''p_m = -200\,\mathrm{fs}^2$, according to Eq. (4) the soliton pulse's bandwidth would be ~7550 GHz, which is much broader than the gain linewidth of 1500 GHz. Consequently, multiple smaller solitonlike spikes with durations comparable to the inverse of the lasing bandwidth are formed. Because of coupling with the low-intensity noise background, these solitonlike spikes are not stable. They exchange energies, collapse, and then regenerate, forming an overall incoherent pulse. When $k''p_m = -2400\,\mathrm{fs}^2$, according to Eq. (4) the soliton pulse's bandwidth would be ~630 GHz, that is, much narrower than the free-running lasing bandwidth of ~1250 GHz. Soliton mode locking fails, resulting in the generation of an incoherent pulse. In both unstable GVD regions the envelope duration of the incoherent pulse is shaped by amplitude modulation and is proportional to the lasing bandwidth.[9] In the small negative GVD region the pulse duration is nearly constant, whereas in the large negative GVD region the envelope of the incoherent pulse is broadened with increasing GVD.[6]

Evidently, parameters such as the gain medium's degree of inhomogeneity, gain linewidth, and SPM affect the dynamics of pulse shaping. In what follows, we discuss the influence of various parameters on soliton stability conditions.

B. Gain Medium's Degree of Inhomogeneity

We consider two laser gain media, both of which have the same inhomogeneous linewidth, 1440 GHz. One gain medium has a homogeneous linewidth $\Delta\nu_{h1} = 360\,\mathrm{GHz}$, so the degree of inhomogeneity is 4. The other one has a homogeneous linewidth $\Delta\nu_{h2} = 850\,\mathrm{GHz}$, so the degree of inhomogeneity is 1.7. The convoluted unsaturated gain linewidths of both gain media are almost the same, ~1500 GHz. Unsaturated gains are adjusted to give the same intracavity energy. Figure 4 compares the mode-locking performance of these two inhomogeneously broadened lasers.

For the laser with the degree of inhomogeneity of 4, a single solitonlike pulse can be produced when the GVD is -800 to $-1800\ \mathrm{fs}^2$. When the gain medium's degree of inhomogeneity becomes 1.7, the GVD region for the formation of a single, clean solitonlike pulse shifts to -2000 to $-10\,000\ \mathrm{fs}^2$. Given the same unsaturated gain linewidth, if the gain medium is less inhomogeneously broadened, the saturated gain linewidth is narrower and the gain filtering is stronger. Consequently, a larger negative GVD is needed to support a soliton pulse. As an extreme case, a purely homogeneously broadened laser with the same unsaturated gain linewidth of 1500 GHz has a lower stability GVD boundary of $\sim -30\,000\ \mathrm{fs}^2$, much greater than that for the inhomogeneously broadened lasers.

The laser with a degree of inhomogeneity of 4 has a free-running bandwidth of ~1250 GHz. For the laser with the degree of inhomogeneity of 1.7, the free-running spectrum has two narrow peaks with a width of ~50 GHz and a separation of ~430 GHz. Thus it is easier for the less inhomogeneously broadened laser to suppress the nonsoliton spectral components, resulting in a larger value of the upper stability boundary for the GVD. By contrast, for a purely homogeneously broadened laser there is no upper stability boundary for the GVD.

The inhomogeneously broadened laser seems to be more appropriate for generating stable and shorter soliton pulses than is the homogeneously broadened laser with active mode locking. A stable GVD range from -800 to $-1800\ \mathrm{fs}^2$ can easily be achieved by use of a pair of prisms or chirped mirrors. However, it is quite difficult to obtain $k''p_m \approx -30\,000\,\mathrm{fs}^2$.

C. Gain Linewidth

To quantify the effect of a limited gain linewidth, we consider two gain media with the same degree of inhomogeneity of 4. One medium has a homogeneous linewidth $\Delta\nu_{h1} = 360\,\mathrm{GHz}$ and the other one has homogeneous linewidth $\Delta\nu_{h2} = 720\,\mathrm{GHz}$, resulting in unsaturated gain linewidths of 1500 and 3000 GHz, respectively. Figure 5 compares the soliton stability ranges for the two lasers. For the laser with the doubled gain linewidth, the cw las-

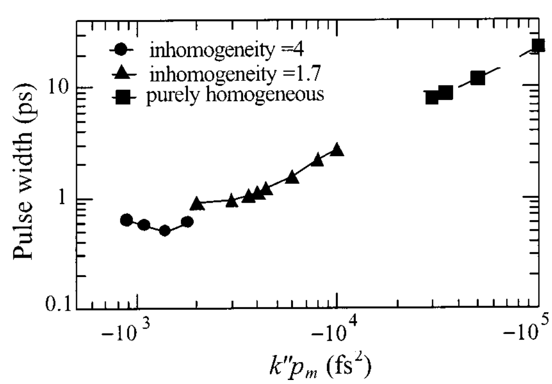

Fig. 4. Regions of GVD for generation of single, stable, solitonlike pulses in lasers with several degrees of inhomogeneity.

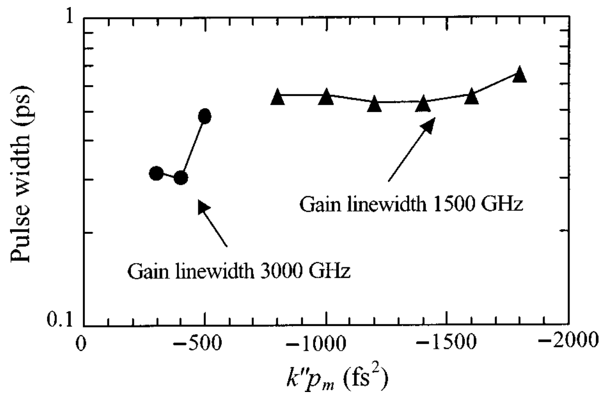

Fig. 5. Stable negative GVD range for lasers with several gain linewidths, given the same degree of inhomogeneity of 4.

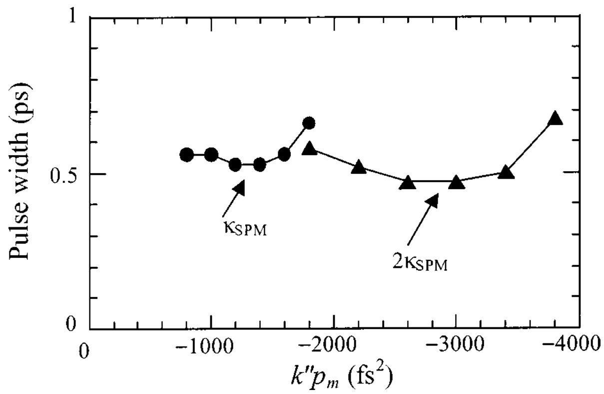

Fig. 6. Stable negative GVD range for lasers with several SPM strengths. The gain medium's degree of inhomogeneity is 4. SPM coefficient $\kappa_{SPM} = 1.88 \times 10^{-5}$ cm^2/MW.

ing bandwidth is also nearly doubled, to ~2500 GHz. As a result of weaker gain filtering, the whole stable soliton regime shifts down to a smaller negative GVD region of -300 to -500 fs^2, and shorter pulses are generated. We notice that the upper stability GVD boundary decreases by more than one half. A decrease of GVD by one half would be expected from a doubled cw lasing bandwidth. The further reduction occurs because as the GVD decreases, the free-running components disperse more slowly and are removed less effectively by the active modulator. As a result, the soliton pulse energy at -900 fs^2 is actually smaller than that at -1800 fs^2, and the soliton pulse's bandwidth is not broad enough to compete for gain with the nonsoliton components. The soliton pulse is thus not stable at -900 fs^2, and the stable soliton region has to move to an even smaller GVD. Thus, increasing the gain linewidth results in a trade-off between generating a shorter solitonlike pulse and having a narrower stable GVD range.

D. Self-Phase Modulation

The soliton pulse's bandwidth is determined by the GVD and SPM. A variation of SPM will thus affect the stable GVD range. Figure 6 compares the soliton stability region with several SPM strengths for the same laser gain medium ($\Delta\nu_h = 360$ GHz; $\Delta\nu_{ih} = 1440$ GHz). The values of both the lower and the upper soliton stability GVD boundaries increase nearly twice for the laser with a doubled SPM coefficient. The reason for the change of stability boundaries is straightforward: Given a fixed intracavity energy and a fixed soliton bandwidth, when the SPM strength doubles, the amount of GVD also doubles. We noticed that the laser can generate a slightly shorter solitonlike pulse with increasing SPM strength. This result is consistent with the observation that with a larger amount of GVD, the low-intensity noise background is suppressed more completely. Therefore the solitonlike pulse receives more energy and becomes shorter. Hence, by increasing the SPM, one can achieve both a shorter solitonlike pulse and a larger stability range.

3. EXPERIMENTAL RESULTS

We experimentally studied actively mode-locked, inhomogeneously broadened Nd:silicate glass and Nd:phosphate glass lasers. The laser cavity used in the experiment was similar to the setup described in Ref. 6. The laser was modulated by an acousto-optic modulator. A pair of SF-10 prisms was used to adjust the intracavity GVD. The cavity length was ~1.6 m.

The lasing transition of Nd:glass consists of multiple Stark-split subtransitions. The observed fluorescence linewidths of Nd:silicate glass and Nd:phosphate glass, in the range 25–30 nm, are from the convolution of homogenous broadening, inhomogeneous broadening, and Stark splitting. For a single Stark-split subtransition, the Nd:silicate glass has a degree of inhomogeneity of ~4,[10] and the Nd:phosphate glass has a degree of inhomogeneity of 1–2.[11] In the experiment, to focus on the GVD effect we purposely did not flatten the gain profile.[5] The pulse bandwidth was relatively narrow, and a single Stark-split subtransition dominated in the lasing operation.[12] Figure 7 shows the free-running spectra of the Nd:silicate glass laser and the Nd:phosphate glass laser, which have lasing bandwidths of ~5 and ~0.7 nm (corresponding to ~1330 and ~240 GHz), respectively. The much narrower

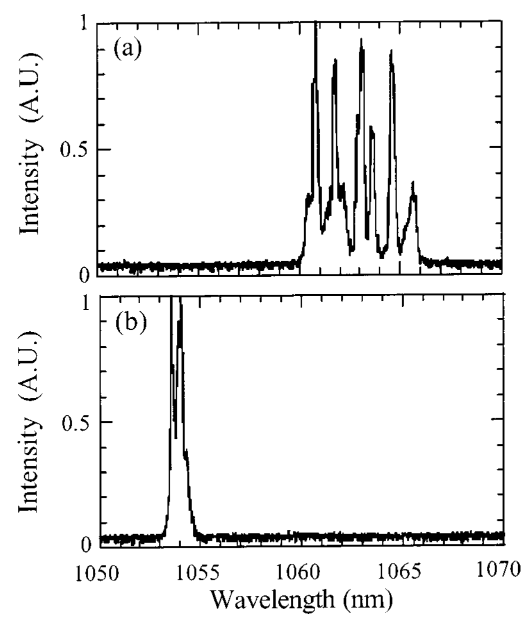

Fig. 7. Free-running spectra of (a) a Nd:silicate glass laser and (b) a Nd:phosphate glass laser.

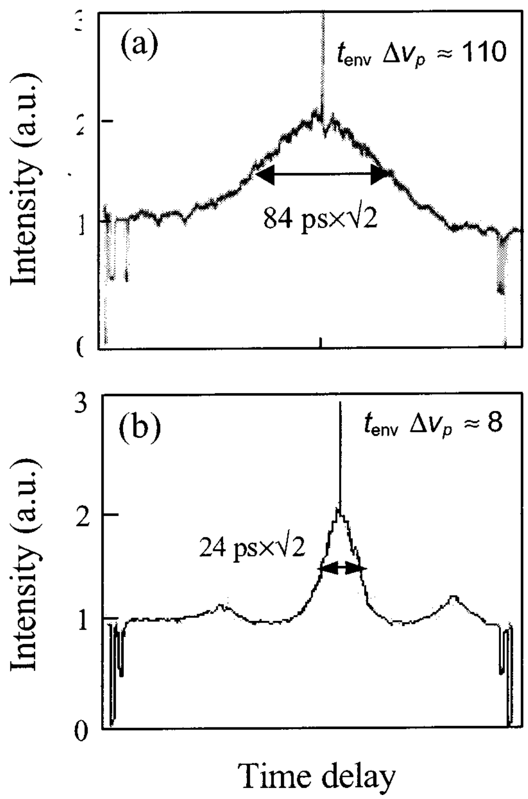

Fig. 8. Pulse autocorrelation traces for (a) a Nd:silicate glass laser and (b) a Nd:phosphate glass laser with $k''p_m \approx -200\ \mathrm{fs}^2$.

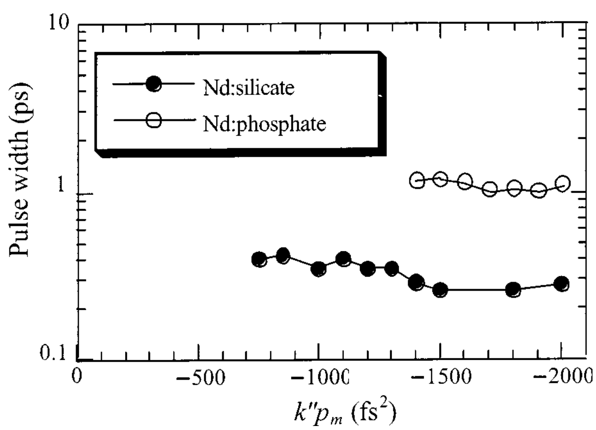

Fig. 9. Comparison of pulse widths of a single, stable soliton pulse in actively mode-locked Nd:silicate glass and Nd:phosphate glass lasers.

cw lasing bandwidth of the Nd:phosphate glass laser confirms that it is more nearly homogeneous.

In the mode-locking experiment we operated the two lasers with approximately the same intracavity energy. In the small negative GVD region, both the Nd:silicate glass laser and the Nd:phosphate glass laser produced only incoherent pulses. The autocorrelation traces are shown in Fig. 8, with $k''p_m \approx -200\ \mathrm{fs}^2$. The Nd:silicate glass laser generated long pulses of poor coherence, with a time–bandwidth product of ~110. The Nd:phosphate glass laser generated pulses with a lower degree of incoherence, and the time–bandwidth product was ~8. In the unstable GVD regions, a broader lasing bandwidth would result in a longer envelope duration and poorer pulse coherence because the envelope of an incoherent pulse is shaped mainly by active amplitude modulation.[8] The experimental results agree with this dependence. Figure 9 compares the GVD range within which a single and stable soliton pulse was generated for the Nd:silicate glass and Nd:phosphate glass lasers. For the Nd:silicate glass laser the minimum negative GVD was ~−750 fs^2. For the more nearly homogeneous Nd:phosphate glass laser, gain filtering was stronger, and the minimum negative GVD increased to ~−1400 fs^2. This result agrees with the numerical analysis of the dependence of the lower soliton stability GVD boundary on the gain medium's inhomogeneity. In this region the actively mode-locked Nd:silicate glass laser generated coherent pulses of 300–500 fs. Slightly shorter pulses were generated at larger negative GVD because the soliton pulse contained more intracavity energy. The Nd:phosphate glass laser generated ~1-ps pulses. Both lasers generated single-soliton pulses with GVD of as much as ~−2000 fs^2, which was limited by the experimental setup. We noticed that, with the same negative GVD, longer pulses were generated from the Nd:phosphate glass laser than from the Nd:silicate glass laser. Two factors contributed to this result. One is that the nonlinear index of refraction for the Nd:phosphate glass is ~40% smaller than that of the Nd:silicate glass. The smaller κ_{SPM} of the Nd:phosphate glass shifts the lower stability boundary to a smaller GVD. In addition, near the lower stability GVD boundary, relatively more intracavity energy is transferred to the low-intensity noise background, resulting in a broader soliton pulse.

4. SUMMARY

We have studied the soliton stability conditions in actively mode-locked, inhomogeneously broadened lasers. In the small negative GVD region the laser generates multiple solitonlike spikes because of excessive gain filtering. In the large negative GVD region the soliton pulse's bandwidth is not broad enough to suppress the nonsoliton spectral components because there is insufficient gain filtering, and the laser generates only incoherent pulses. The instability in the large negative GVD region occurs only for inhomogeneously broadened lasers. When the laser gain medium is more inhomogeneously broadened or for a broader unsaturated gain linewidth, the gain filtering effect is weaker, and both the lower and the upper stability boundaries shift to small negative GVD. A larger SPM, however, changes both boundaries toward more-negative GVD, and a broader stable GVD range results. The experimental results with actively mode-locked Nd:silicate glass and Nd:phosphate glass lasers confirm such a GVD requirement for the generation of a single, stable soliton pulse and confirm the dependence of the lower soliton stability GVD boundary on the gain medium's inhomogeneity.

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