Influence of the Model for Random Birefringence on the Differential Group Delay of Periodically Spun Fibers

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***Abstract*—We consider the two Wai–Menyuk models of birefringence in periodically spun fibers, and we show that the differential group delay differs significantly for the two models when the spin period approaches or exceeds the fiber beat length. When the fiber correlation length is large, we explain this difference quantitatively, and we explain it qualitatively for any fiber correlation length.**

***Index Terms*—Differential group delay (DGD), fiber birefringence, polarization mode dispersion (PMD), spun fiber.**

I. Introduction

SPUN FIBERS were first used in sensors about 20 years ago, but in more recent years they have been increasingly used in telecommunication systems to reduce polarization mode dispersion (PMD), [1]–[4]. The authors of [2]–[4] mainly investigated the effects of periodic spin functions in the "short-period" limit, i.e., the regime in which the spin period p is shorter than the beat length L_B. Moreover in [2] and [3], the authors assumed that the fiber birefringence is deterministic, corresponding to an infinite correlation length L_F. In [4], the birefringence was random, as is the case in real telecommunications fibers and was modeled with a fixed strength and varying orientation, according to the first physical model [fixed modulus model (FMM)] proposed by Wai and Menyuk in [5].

However, we remark that many experiments have shown that the birefringence strength is not fixed, but instead, it varies at random [6], validating the second model proposed in [5] [random modulus model (RMM)]. Therefore, it is important to study the behavior of randomly birefringent spun fibers according to the RMM and to understand the differences between the two models. The authors of [4] showed that their results were consistent with the RMM by means of numerical simulations in the short-period limit, but it is important to investigate all regimes.

In this letter, we show that when the spin period is of the same order of magnitude as the beat length, spun fibers which obey the two different fiber models, but which have the same basic fiber parameters, have substantially different values of the mean differential group delay (DGD). This result implies that the two different models will lead to substantially different PMD behavior, in contrast to unspun fibers, where the expected DGD, and hence, the PMD behavior is model-independent.

After a brief description of the two birefringence models, we report a comparison between them in the regime $p \simeq L_B$, obtained by means of Monte Carlo simulations. Finally, we provide a physical explanation for the difference between the two models in computing the DGD of a spun fiber. We stress that the use of the simpler FMM may lead to results that do not reproduce a real fiber's behavior when the fiber is spun.

II. Theoretical Background

PMD characteristics are completely described by the polarization dispersion vector $\mathbf{\Omega}(z, \omega)$ whose modulus is equal to the DGD $\Delta\tau$ [7]. The evolution of $\mathbf{\Omega}$ as a function of distance is governed by the dynamical equation [8]

$$\frac{\partial \mathbf{\Omega}(z, \omega)}{\partial z} = \frac{\partial \boldsymbol{\beta}(z, \omega)}{\partial \omega} + \boldsymbol{\beta}(z, \omega) \times \mathbf{\Omega}(z, \omega) \qquad (1)$$

where $\boldsymbol{\beta}(z, \omega)$ is the local birefringence vector.

Both the FMM and the RMM assume that fibers are linearly birefringent. Then, according to the FMM, the linear local birefringence vector $\boldsymbol{\beta}_l$ may be written $\boldsymbol{\beta}_l = [b\cos\theta(z), b\sin\theta(z), 0]^T$, where $b = 2\pi/L_B$ is the fixed birefringence strength, $\theta(z)$ is the birefringence orientation that varies with rate so that $d\theta/dz = \eta(z)$, and $\eta(z)$ is a Gaussian white noise process with zero mean and variance $\sigma_\eta^2 = 2/L_F$. Alternatively, according to the RMM, the two components of the local birefringence of the unspun fiber are independent Langevin processes [5]

$$\frac{d\beta_i}{dz} = -\rho\beta_i(z) + \sigma\eta_i(z), \qquad i = 1, 2 \qquad (2)$$

where $\eta_1(z)$ and $\eta_2(z)$ are independent, delta-correlated white noise processes with zero mean and $\rho = 1/L_F$. With a suitable choice of the initial condition, β_1 and β_2 are independent wide-sense-stationary Gaussian processes with zero mean and variances $\langle\beta_1^2\rangle = \langle\beta_2^2\rangle = \sigma_\beta^2 = \sigma^2/2\rho$. Therefore, the modulus

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of birefringence β_l is a Rayleigh random variable with a probability density function

$$f(\beta_l) = \frac{\beta_l}{\sigma_\beta^2} \exp\left(-\frac{\beta_l^2}{2\sigma_\beta^2}\right) \tag{3}$$

and the first two statistical moments are

$$\langle \beta_l \rangle = \sigma_\beta \sqrt{\frac{\pi}{2}}, \qquad \langle \beta_l^2 \rangle = \frac{4\pi^2}{L_B^2} = \frac{4}{\pi} \langle \beta_l \rangle^2. \tag{4}$$

For both models, after introducing the assumption that $d\beta_l/d\omega = \beta_l/\omega$, the mean square DGD of the unspun fiber, is [5]

$$\langle \Delta\tau_{\mathrm{un}}^2 \rangle = \frac{2L_F^2}{\omega^2} \langle \beta_l^2 \rangle \left[\exp\left(-\frac{z}{L_F}\right) + \frac{z}{L_F} - 1\right]. \tag{5}$$

Finally, when a fiber is spun according to a spin function $A(z)$, no circular birefringence is induced and the only effect is a rotation of the local birefringence vector [1]

$$\boldsymbol{\beta}(z, \omega) = \begin{pmatrix} \cos 2A(z) & -\sin 2A(z) & 0 \\ \sin 2A(z) & \cos 2A(z) & 0 \\ 0 & 0 & 1 \end{pmatrix} \boldsymbol{\beta}_l(z, \omega). \tag{6}$$

III. Simulation Results

In order to investigate the spin effects when the short-period assumption is not satisfied, we implement the FMM and the RMM and perform Monte Carlo simulations, solving (1), using the wave-plate model for a set of 6000 fibers for each of the two birefringence models. We choose a sinusoidal spin function $A(z) = A_0 \sin(2\pi z/p)$ with a period $p = 4$ m and a wave-plate length equal to 10 mm. We fix the beat length $L_B = 5$ m, while we vary the spin amplitude A_0 and the correlation length L_F. For each value of L_F, we consider a fiber length $z > 100\,L_F$ so that the transient behavior has completely died out, and we calculate the *spin-induced reduction factor* (SIRF), i.e., the ratio between the mean DGD of the spun and unspun fiber [4]. We remark that the results that we present are qualitatively similar to those when $p > L_B$, even though they have been obtained in the case $p \simeq L_B$.

Fig. 1(a) shows the SIRF as a function of the spin amplitude obtained using the FMM. The curves, from the upper curve to the lower curve, refer to $L_F = 0.5$, 1, 3, 16, and 50 m, respectively. For small values of L_F, the spin is not very effective in reducing the mean DGD because the intrinsic random fluctuations of birefringence are faster, but as soon as L_F increases, the reduction factor shows the same behavior as in the limit where the period is short [4]. In particular, for $L_F \to +\infty$, there exist values of A_0 for which the reduction factor vanishes, although such resonant amplitudes do not coincide with those predicted in [2] and [4], since the period is not short. Moreover, when $L_F = 16$ m, the SIRF is already very close to the deterministic limit.

For comparison, Fig. 1(b) reports the SIRF for the same spin and fiber parameters, but obtained with the RMM. There is agreement between the two models for small values of L_F, but when L_F increases, the SIRF obtained with the RMM is substantially different from the one estimated with the FMM. In particular, when $L_F \to +\infty$, we find with the RMM that there are spin amplitude values that correspond to relative minima, but the SIRF never goes to zero. Moreover, the positions of the relative minima do not coincide with those obtained using the FMM. Hence, the prediction of mean DGD by means of the FMM leads to incorrect results when the spin period and beat length are of the same order, and, in particular, one can overestimate the benefit of spinning in reducing the mean DGD of the fiber.

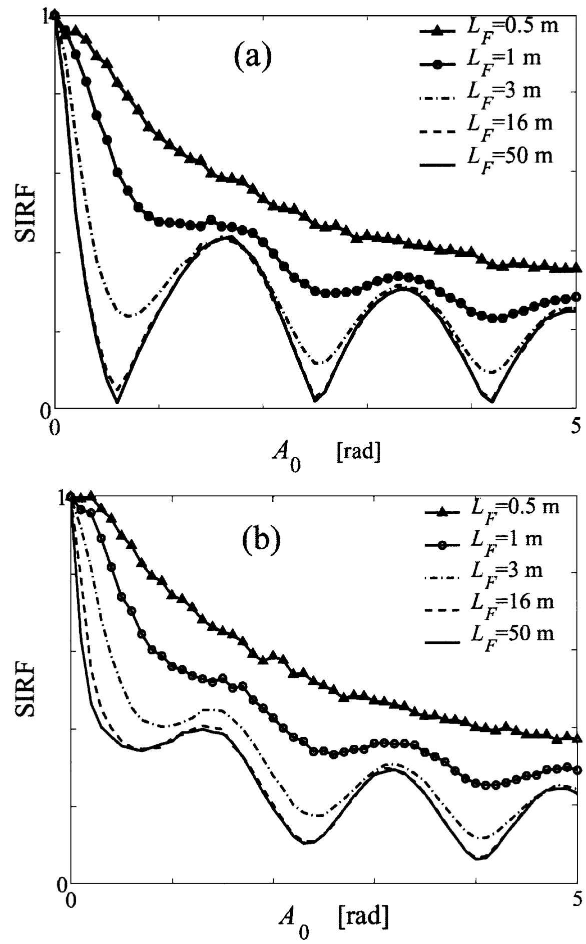

Fig. 1. Evolution of the SIRF as a function of the spin amplitude A_0 for a sinusoidal spin function with period $p = 4$ m and with $L_B = 5$ m. The different curves, from the upper to the lower correspond to $L_F = 0.5$, 1, 3, 16, and 50 m, respectively. Plots (a) and (b) correspond to the FMM and the RMM, respectively.

IV. Explanation of the Difference

The key to understanding the difference between Fig. 1(a) and (b) is that in the FMM the modulus of the birefringence is fixed, while in the RMM it varies randomly along each fiber realization. It is easiest to understand the impact of the varying birefringence in the limit $L_F \gg p$, while keeping the fiber length $z \gg L_F$. In this limit, each fiber realization can be viewed as a concatenation of long fiber sections, in which the length is of order L_F and the birefringence is constant. Since an ergodic

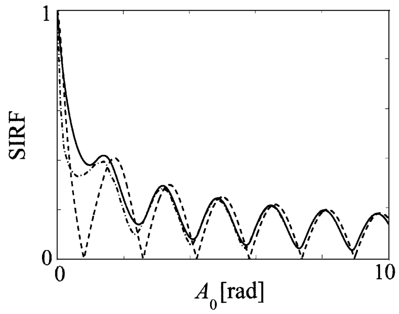

Fig. 2. Evolution of the SIRF as a function of the spin amplitude A_0 for $L_B = 5$ m and $L_F = 50$ m. The dashed line is obtained with the FMM. The solid line corresponds to the RMM and is obtained by weighting the FMM data according to the Rayleigh distribution of the local birefringence. The dashed–dotted line is the same reported in Fig. 1(b) for $L_F = 50$ m.

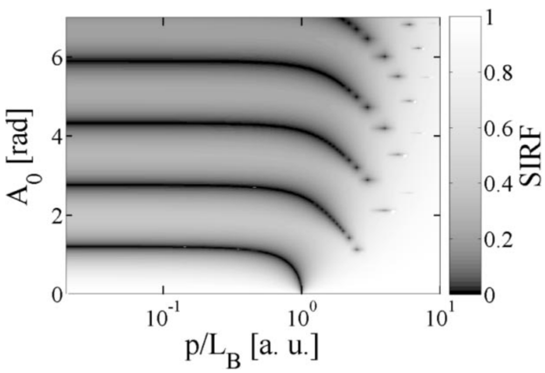

Fig. 3. Evolution of the SIRF as a function of the spin amplitude A_0 and of the ratio p/L_B in the case $L_F = 50$ m for the FMM.

theorem holds for an ensemble of optical fibers, so that the ensemble average is equivalent to a long spatial average over a single realization [5], [9], it follows that the SIRF in the RMM must equal the sum of the SIRFs for each different birefringence modulus in the FMM, weighted by its probability of occurring. This probability, given by (3), is Rayleigh-distributed. The result is to smooth out the SIRF and to eliminate the zeros. In Fig. 2, we compare the SIRF for the FMM and the RMM with $p = 4$ m and $L_B = 5$ m for $L_F = 50$ m. The smoothing is clearly visible. When L_F is small, a similar smoothing is expected to occur. However, there is no longer a simple relationship between the FMM and the RMM.

Why then is there no difference between the FMM and the RMM in the short-period limit? The explanation is that the SIRF is independent of the fiber beat length in the short-period limit as shown in [2] and [4]. To better understand this point we calculate the SIRF for the first model of birefringence for $L_F = 50$ m as a function of the spin amplitude A_0 and of the ratio p/L_B. We show the result in Fig. 3, where the gray scale corresponds to different SIRF values, so that black refers to SIRF = 0. In Fig. 3, we use the sinusoidal spin function $A(z) = A_0 \sin(2\pi z/p)$; we then fix $p = 4$ m and vary L_B from 0.3 m up to approximately 300 m. The horizontal axis shows the ratio p/L_B on a logarithmic scale. One sees that when $p < L_B$, the zones corresponding to any given value of the SIRF are straight lines parallel to the horizontal axis, confirming their independence of L_B.

Thus, since the SIRF in the FMM is independent of L_B, when we average over the different values of L_B in the FMM to obtain the RMM, we obtain exactly the same curve that we started out with. By contrast, if $p \gtrsim L_B$, then the SIRF zeros or any other fixed SIRF values are no longer parallel to the horizontal axis, indicating that the SIRF in the FMM now depends on L_B. Thus, we obtain the smoothing effect that is visible in Fig. 2.

V. Conclusion

We have shown that the effect of spin on the fiber DGD depends significantly on the model that is assumed for the fiber's local birefringence when $p \gtrsim L_B$. This behavior contrasts strongly with unspun fibers, in which the PMD is model-independent. In particular, the use of the FMM can overestimate the benefit of spinning in reducing PMD. In the limit $L_F \gg p$, we have shown that it is possible to calculate the DGD in the RMM as an appropriate average over the SIRF of the FMM at different values of L_B, resulting in a smoothing of the SIRF function. When L_F is small, we have found numerically that a similar smoothing is present. Finally, the model dependence of the DGD has strong implications for the effectiveness of spinning in minimizing fiber PMD. The analysis presented in this letter shows that previously developed techniques for prediction of fiber PMD lead to correct results only in the case $p < L_B$ because they have been derived for fixed birefringence or for the FMM, while the birefringence of real fibers is closer to the RMM.

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