Polarization mode dispersion of spun fibers with randomly varying birefringence

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We show analytically how periodic spinning affects the polarization mode dispersion of a fiber in three different practical regimes that are determined by the values of three length scales: the beat length, the birefringence correlation length, and the spin period. We determine in which limits the spin is effective in reducing the mean differential group delay. © 2003 Optical Society of America

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Polarization mode dispersion (PMD) can be effectively reduced by spinning of a fiber during the drawing process. Several papers that focused on this topic were published recently.[1–3]

The PMD of randomly birefringent unspun fibers is characterized by two length scales: the beat length, L_B, that is inversely proportional to the mean birefringence strength, and the birefringence correlation length, L_F, that describes the length scale over which an ensemble of fibers with randomly varying birefringence becomes uncorrelated. When a fiber is spun periodically, two other quantities come into play: the spin period, p, and the spin amplitude, A_0.

A spin function $A(z)$ corresponds to applying a torque while the fiber is still at a temperature close to the silica melting point. Thus a spin does not induce torsional stresses, and its effect is to rotate the orientation of fiber birefringence. Intuitively, if one periodically exchanges the fast and slow axes, it should be possible to cancel out the differential group delay (DGD), at least when the beat length is long compared with the spin period such that the polarization state of the light does not evolve significantly in one spin period.

The authors of Refs. 1–3 focused mainly on the spin effects in the short-period limit, in which the spin period is much shorter than the beat length. Only in Ref. 3 was random birefringence considered, and it was modeled with a fixed strength and varying orientation, according to one of the two physical models proposed by Wai and Menyuk.[4]

However, it is difficult in general to predict the beat length and the correlation length of a fiber. Moreover, measurements performed to date[5] show that both L_B and L_F can vary over a wide range of values, from approximately 1 m to tens of meters, depending on the fiber type and whether it is wound on a bobbin or deployed in a cable. Thus it is of interest to characterize the behavior of spun fibers when the short-period assumption is not satisfied and to understand in which regimes the spin is effective in reducing the mean DGD.

In this Letter we consider three asymptotic limits. We show by means of perturbation techniques[6] how the introduction of a periodic spin influences the PMD of a fiber in the different regimes when one of the three length scales is much shorter than the others.

We begin by considering a model of birefringence[4] in which the magnitude of the birefringence is constant. If one considers a rotating reference frame that compensates for the rotations that are due to both the intrinsic birefringence and the spin, the local birefringence vector is $\beta = [b,\, 0, -2\alpha(z) + \sigma\eta(z)]^T$, where $b = 2\pi/L_B$ is the fixed birefringence strength, $\alpha(z)$ is the z derivative of spin function $A(z)$, $\eta(z)$ is a white-noise process that drives the random variation of the birefringence axes, and $2\sigma^2 = 1/L_F$. Exploiting the theory of stochastic differential equations,[7] we can determine the evolution of the mean-square DGD, which reads as[3]

$$\frac{\mathrm{d}\langle \Delta\tau^2(z)\rangle}{\mathrm{d}z} = 2b_\omega \langle \Omega_1(z)\rangle\,, \tag{1}$$

where b_ω is the frequency derivative of b and $\Omega_1(z)$ represents the first component of $\Omega(z)$, the polarization dispersion vector. Similarly, we find the following system of differential equations[3]:

$$\begin{aligned}
\frac{\mathrm{d}\langle \Omega_1\rangle}{\mathrm{d}z} &= -2\sigma^2\langle\Omega_1\rangle + 2\alpha(z)\langle\Omega_2\rangle + b_\omega\,,\\
\frac{\mathrm{d}\langle \Omega_2\rangle}{\mathrm{d}z} &= -2\alpha(z)\langle\Omega_1\rangle - 2\sigma^2\langle\Omega_2\rangle - b\langle\Omega_3\rangle\,,\\
\frac{\mathrm{d}\langle \Omega_3\rangle}{\mathrm{d}z} &= b\langle\Omega_2\rangle\,.
\end{aligned} \tag{2}$$

In general, it is possible to solve Eqs. (2) only numerically; however, we still analytically calculate their asymptotic solution in different limits.

Because of the relative scaling of the quantities of interest, we introduce a useful change of notation. If we consider a spin function with period p and amplitude A_0, then spin rate $\alpha(z)$ is proportional to the ratio $2\pi A_0/p$, so we write $\alpha(z) = A_0\nu q(z)$, where $|q(z)|$ is of order 1, and $\nu = 2\pi/p$, is the spatial frequency. Thus, in what follows, we calculate the effect of spinning on the mean DGD, depending on the relative magnitudes of $A_0\nu$, $2\sigma^2$, and b. We use a straightforward perturbation expansion.[6] Although straightforward expansions frequently can lead to a nonuniform asymptotic

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expansion,[6] we have verified that all the asymptotic expansions reported in this Letter for the mean DGD are uniform.

In the first case that we study, we explore the limit when the spin period is much shorter than the other two length scales, i.e., $A_0\nu \gg 2\sigma^2$ and $A_0\nu \gg b$. Under these conditions we expand the solution of Eqs. (2) by assuming that all terms that do not contain $\alpha(z)$ drop out at lowest order, except for the inhomogeneous term, which is required for nonzero values of Ω_i to be obtained. Then the averaging brackets may be removed, and the resultant equations may be integrated by the method of variation of parameters along with the initial conditions $\Omega_j = 0$ for $j = 1, 2, 3$. By substituting

$$\begin{aligned}\Omega_1(z) &= k_1(z)\sin[2A(z)] + k_2(z)\cos[2A(z)],\\ \Omega_2(z) &= -k_2(z)\sin[2A(z)] + k_1(z)\cos[2A(z)],\end{aligned} \quad (3)$$

we find that

$$\begin{aligned}k_1 &= b_\omega \int_0^z \sin[2A(z')]\mathrm{d}z' \equiv b_\omega S\,,\\ k_2 &= b_\omega \int_0^z \cos[2A(z')]\mathrm{d}z' \equiv b_\omega C\,,\end{aligned} \quad (4)$$

from which we conclude that

$$\Delta\tau^2 = b_\omega{}^2(C^2 + S^2) = b_\omega{}^2 \left| \int_0^z \exp[i2A(z')]\mathrm{d}z' \right|^2 . \quad (5)$$

The agreement of $\Delta\tau^2$ predicted by Eq. (5) with $\Delta\tau^2$ reported in Ref. 2 and also with $\Delta\tau^2$ that might be calculated following the procedure explained in Ref. 1 confirms the validity of the procedure that we followed. Additionally, Eq. (5) is obtained in a more straightforward and general way.

Next, we consider the case when L_F is shorter than $p/(2\pi A_0)$, such that, in terms of spatial frequencies, $2\sigma^2 \gg A_0\nu$. To apply the perturbation technique it is necessary also to fix the relationship between $A_0\nu$ and b. The expansion that we perform holds in both cases, $A_0\nu \simeq b$ and $A_0\nu \gg b$ (or $p/A_0 \simeq L_B$ and $p/A_0 \ll L_B$), that are of practical interest because the spinning is expected to be more effective when its period is shorter than the beat length. So, considering a spin amplitude of a few radians, this limit may be written as $L_F \ll (p, L_B)$. Under these conditions, we obtain the leading-order solution by solving the system

$$\frac{\mathrm{d}\langle\Omega_1\rangle^{(0)}}{\mathrm{d}z} = -2\sigma^2\langle\Omega_1\rangle^{(0)} + b_\omega\,,$$
$$\frac{\mathrm{d}\langle\Omega_2\rangle^{(0)}}{\mathrm{d}z} = -2\sigma^2\langle\Omega_2\rangle^{(0)}, \qquad \frac{\mathrm{d}\langle\Omega_3\rangle^{(0)}}{\mathrm{d}z} = 0\,. \quad (6)$$

The solution of Eqs. (6) is that of the unspun fiber: $\langle\Omega_1\rangle^{(0)} = b_\omega/2\sigma^2[1 - \exp(-2\sigma^2 z)]$, $\langle\Omega_2\rangle^{(0)} = \langle\Omega_3\rangle^{(0)} = 0$. Then the first-order correction is the solution of the system

$$\begin{aligned}\frac{\mathrm{d}\langle\Omega_1\rangle^{(1)}}{\mathrm{d}z} &= -2\sigma^2\langle\Omega_1\rangle^{(1)} + 2\alpha(z)\langle\Omega_2\rangle^{(0)},\\ \frac{\mathrm{d}\langle\Omega_2\rangle^{(1)}}{\mathrm{d}z} &= -2\sigma^2\langle\Omega_2\rangle^{(1)} - 2\alpha(z)\langle\Omega_1\rangle^{(0)} - b\langle\Omega_3\rangle^{(0)},\\ \frac{\mathrm{d}\langle\Omega_3\rangle^{(1)}}{\mathrm{d}z} &= b\langle\Omega_2\rangle^{(0)}.\end{aligned} \quad (7)$$

Consequently, at this order, there is no correction to the first and third components of the average polarization dispersion vector. While solving the second of Eqs. (7) for $\langle\Omega_2\rangle^{(1)}$, we introduce the spin function $A(z) = A_0 \sin(\nu z)$ and obtain

$$\begin{aligned}\langle\Omega_2\rangle^{(1)} = -\frac{A_0 b_w}{\sigma^2(4\sigma^4 + \nu^2)}\{&2\sigma^2\nu\cos(\nu z) + \nu^2\sin(\nu z)\\ &-\exp(-2\sigma^2 z)[2\sigma^2\nu + (4\sigma^4 + \nu^2)\sin(\nu z)]\}.\end{aligned} \quad (8)$$

The equation for the second-order correction has the same form as Eqs. (7). After substitution of Eq. (8) into the equation for $\langle\Omega_1\rangle^{(2)}$, it follows that

$$\begin{aligned}\langle\Omega_1\rangle^{(2)} \simeq -\frac{A_0{}^2\nu^2 b_w}{\sigma^2(4\sigma^4 + \nu^2)(\sigma^4 + \nu^2)}\{&\sigma^4[1 + \cos(2\nu z)]\\ &+\nu^2 + \sigma^2\nu\sin(2\nu z) - (2\sigma^4 + \nu^2)\exp(-2\sigma^2 z)\},\end{aligned} \quad (9)$$

which is simplified under the condition that $z \gg L_F$. We also find that $\langle\Omega_2\rangle^{(2)} = \langle\Omega_3\rangle^{(2)} = 0$. Then the sum $\langle\Omega_1\rangle^{(0)} + \langle\Omega_1\rangle^{(2)}$ can be integrated over a distance to yield the mean-square DGD, according to Eq. (1), yielding the following simple expression:

$$\langle\Delta\tau^2\rangle \simeq \langle\Delta\tau_{\mathrm{un}}{}^2\rangle\left(1 - \frac{2A_0{}^2\nu^2}{4\sigma^4 + \nu^2}\right), \quad (10)$$

where $\langle\Delta\tau_{\mathrm{un}}{}^2\rangle$ is the mean-square DGD of the corresponding unspun fiber. Relation (10) is always positive in the limit $2\sigma^2 \gg A_0\nu$, in which the derivation is carried out. To better estimate the limits of validity of relation (10) we compared it to the complete numerical integration of Eqs. (2), and we calculated the relative error of the mean DGD. Figure 1 shows the relative error, expressed in percent, as a function of the spin amplitude for several values of beat length, spin period, and correlation length. Note that the error is bigger when $p \simeq L_B$ (dashed–dotted curve), in which case the error is less than 5% if $p/(2\pi A_0) \geq 6L_F$. Conversely, when $p \leq L_B$ we find that $p/(2\pi A_0)$ does not have to be as large as $6L_F$ for the error to be less than 5%.

Relation (10) yields another important piece of information. In this regime the DGD of a spun fiber is not strongly affected by the spinning. For example, when $p/(2\pi)A_0 = 6L_F$ with $A_0 = 1$, we find that $\langle\Delta\tau\rangle \simeq 0.9\langle\Delta\tau_{\mathrm{un}}\rangle$.

Finally, we explore the limit of large birefringence, i.e., $b \gg A_0\nu$ and $b \simeq 2\sigma^2$, or, in terms of length scales $p/A_0 \gg L_B$ and $L_B \simeq L_F$. From a physical point of view, a large birefringence corresponds to a rapid rotation on the Poincaré sphere. This effect can be treated more easily if the following transformation is made[8]:

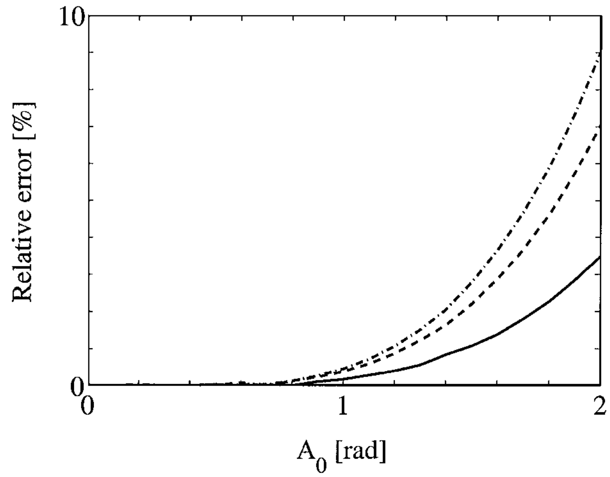

Fig. 1. Evolution of the relative error of the mean-square DGD made with relation (10) instead of the complete solution of Eqs. (2) as a function of spin amplitude, in the limit of short correlation length. Solid curve, $L_B = 24$ m, $p = 18$ m, and $L_F = 0.5$ m. Dashed curve, $L_B = 20$ m, $p = 15$ m, and $L_F = 0.5$ m. Dashed–dotted curve, $L_B = 15$ m, $p = 14$ m, and $L_F = 0.5$ m.

$$\langle \mathbf{y} \rangle = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(bz) & \sin(bz) \\ 0 & -\sin(bz) & \cos(bz) \end{bmatrix} \langle \boldsymbol{\Omega} \rangle . \qquad (11)$$

Because Eq. (11) is an orthogonal transformation, it does not affect Eq. (1); i.e., $\mathrm{d}\langle \Delta\tau^2 \rangle/\mathrm{d}z = 2b_\omega \langle y_1 \rangle$. However Eqs. (2) are transformed into

$$\frac{\mathrm{d}\langle y_1 \rangle}{\mathrm{d}z} = b_\omega - 2\sigma^2 \langle y_1 \rangle + 2\alpha(z)[\langle y_2 \rangle \cos(bz) - \langle y_3 \rangle \sin(bz)],$$

$$\frac{\mathrm{d}\langle y_2 \rangle}{\mathrm{d}z} = -\sigma^2 \langle y_2 \rangle - 2\alpha(z) \langle y_1 \rangle \cos(bz) - \sigma^2[\langle y_2 \rangle \cos(2bz) + \langle y_3 \rangle \sin(2bz)],$$

$$\frac{\mathrm{d}\langle y_3 \rangle}{\mathrm{d}z} = -\sigma^2 \langle y_3 \rangle + 2\alpha(z) \langle y_1 \rangle \sin(bz) + \sigma^2[\langle y_3 \rangle \cos(2bz) + \langle y_2 \rangle \sin(2bz)]. \qquad (12)$$

The sinusoidal terms in Eqs. (12) oscillate rapidly, and at lowest order only their average matters.[8] They have zero mean value, and, after introducing the initial condition $\Omega_j = 0$ for $j = 1, 2, 3$, we obtain again the solution for the unspun fiber[4]: $\langle y_1 \rangle^{(0)} = b_\omega/2\sigma^2[1 - \exp(-2\sigma^2 z)]$, $\langle y_2 \rangle^{(0)} = \langle y_3 \rangle^{(0)} = 0$. Then the equations for the first-order correction have the form

$$\frac{\mathrm{d}\langle y_1 \rangle^{(1)}}{\mathrm{d}z} = -2\sigma^2 \langle y_1 \rangle^{(1)},$$

$$\frac{\mathrm{d}\langle y_2 \rangle^{(1)}}{\mathrm{d}z} = -\sigma^2 \langle y_2 \rangle^{(1)} - 2\alpha(z) \langle y_1 \rangle^{(0)} \cos(bz),$$

$$\frac{\mathrm{d}\langle y_3 \rangle^{(1)}}{\mathrm{d}z} = -\sigma^2 \langle y_3 \rangle^{(1)} + 2\alpha(z) \langle y_1 \rangle^{(0)} \sin(bz). \qquad (13)$$

In this case there is also no correction to the first component of the average polarization dispersion vector, whereas we have to solve two similar differential equations for $\langle y_2 \rangle^{(1)}$ and $\langle y_3 \rangle^{(1)}$.

The complete solution for $\langle y_2 \rangle^{(1)}$ and $\langle y_3 \rangle^{(1)}$ is quite complex and is not presented here. However, the second-order correction for $\langle y_1 \rangle$ is

$$\frac{\mathrm{d}\langle y_1 \rangle^{(2)}}{\mathrm{d}z} = -2\sigma^2 \langle y_1 \rangle^{(2)} + 2\alpha(z)[\langle y_2 \rangle^{(1)} \cos(bz) - \langle y_3 \rangle^{(1)} \sin(bz)]. \qquad (14)$$

Equation (14) can be solved under the assumption that $A(z) = A_0 \sin(\nu z)$, and neglecting terms that tend to zero for $z \gg L_F$, we obtain

$$\langle y_1 \rangle^{(2)} \simeq -\frac{2{A_0}^2 \nu^2 b_\omega}{2\sigma^2 b^2}[1 + \cos(2\nu z) - 2\exp(-2\sigma^2 z)]. \qquad (15)$$

The sum $\langle y_1 \rangle^{(0)} + \langle y_1 \rangle^{(2)}$ can then be integrated over distance to produce an approximate expression for the mean-square DGD: $\langle \Delta\tau^2 \rangle \simeq \langle \Delta{\tau_{\mathrm{un}}}^2 \rangle[1 - (2{A_0}^2\nu^2)/b^2]$. This result demonstrates that the spin does not help in reducing PMD when its period is significantly longer than the beat length.

In conclusion, we have analytically calculated the mean DGD of periodically spun, randomly birefringent fibers in three limits of practical interest that depend on the ratio among L_B, L_F, and spin period p. The theoretical results are obtained in a general way and are a proof that spinning is effective in reducing PMD effects only if the spin rate is at least as fast as the evolution of the intrinsic birefringence and random perturbations.

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