Analysis of PMD Compensators With Fixed DGD Using Importance Sampling

I. T. Lima, Jr., G. Biondini, B. S. Marks, W. L. Kath, and C. R. Menyuk

***Abstract*—In this letter, we use importance sampling to analyze polarization-mode dispersion compensators with a constant differential group delay (DGD) element. We optimize the value of the fixed DGD element of the compensator with respect to the outage probability. We show that the optimum value of the fixed DGD element of the compensator can reduce the outage probability by several orders of magnitude, even though it does not provide a substantial reduction of the average penalty due to polarization-mode dispersion in the cases that we studied. By contrast, choosing the fixed DGD element to maximally reduce the average penalty may lead to an outage probability that is orders of magnitude larger than the optimal choice.**

***Index Terms*—Birefringence, compensation, optical communications, optical fiber dispersion, optical fiber polarization.**

I. Introduction

THERE is substantial interest in upgrading the current per channel data rates to 10 Gb/s and beyond in terrestrial wavelength-division-multiplexed (WDM) systems. Polarization-mode dispersion (PMD) is a significant barrier to achieving this goal. There have been numerous proposals to use optical PMD compensators to mitigate this problem [1]–[7], and much of this work has focused on compensators with a single differential group delay (DGD) element because they are the simplest to build, to control, and to analyze. Theoretical [1]–[4] and experimental [5]–[7] studies have shown that PMD compensators with a single DGD element can drastically reduce the average pulse spreading, and hence, the average bit error rate (BER) of optical systems. However, this average reduction does not address the issue of greatest practical importance. Designers want to ensure that the probability of a power penalty due to pulse spreading beyond some value occurs only a very small fraction of the time. For example, a designer might require that a power penalty larger than 1 dB occurs with probability 10^{-6} or less [2]. In this contribution, we show how to use importance sampling [8] to accurately calculate outage probabilities due to PMD [9]–[12]. The random mode coupling in an optical fiber varies with time due to vibrations and variations of the temperature, and we assume that the fiber passes ergodically through all possible polarization states with the same PMD.

In this contribution, we focus on a compensator with a constant DGD element due to its simplicity and practical importance [5]. We find the value of the fixed DGD element of the compensator that most reduces the outage probability.

II. Theory

One of the important characteristics of optical fibers with PMD is that every frequency has two eigenstates, referred to as the principal states of polarization [13], along which one can transmit optical signals without first-order distortion. The two principal states of polarization have a time of flight difference that is defined as the DGD $\Delta\tau$. Thus, amplitude modulated signals launched into fibers with PMD are broadened according to the amount of DGD and according to how the signal is divided between the two principal states of polarization at the channel frequency. Despite the fact that the DGD is a random quantity that is frequency dependent, it has a correlation bandwidth in which there is very little variation of both the DGD and the principal states of polarization as a function of frequency. This correlation bandwidth is approximately $\Delta\nu \approx 0.5/\langle\Delta\tau\rangle$ [13], where $\langle\Delta\tau\rangle$ is the average DGD. Hence, for systems with $\langle\Delta\tau\rangle = 25$ ps and $\langle\Delta\tau\rangle = 30$ ps, the correlation bandwidth is large compared to the optical bandwidth of the 10-Gb/s nonreturn-to-zero (NRZ) system that we are studying. Thus, it is physically reasonable to assume that the PMD-induced penalties are highly correlated with the DGD at the center of the signal's bandwidth. We have validated this assumption in our simulations by direct observations.

In order to compensate for PMD distortions, we use a compensator with an arbitrarily rotatable polarization controller and a fixed DGD element [5]. We note that the parameters of the polarization controller's orientation are the only free parameters that a compensator with a fixed DGD element possesses.

The main idea behind our use of importance sampling is to bias the probability density function (pdf) of the DGD in such a way as to cause large DGD events to occur more frequently. Since, as previously noted, the power penalty prior to correction and the DGD are strongly correlated, this approach allows us to observe the low probability events that lead to an outage. We note that second- and higher order PMD are included in our simulations, but we do not specifically bias our simulations toward large values of the higher order PMD other than the moder-

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ately large values that are naturally obtained when one biases the first-order PMD. For the cases that are presented here, we have verified that the second-order PMD is uncorrelated with the initial penalty, except as would be expected because of the general correlation between the second- and first-order PMD. We have also verified that the variance in our results is low. These observations are strong evidence that very large values of higher order PMD do not play a significant role, and thus, will not lead to inaccuracies in our results. We are currently pursuing studies in which we bias both first- and second-order PMD to further clarify this point.

To apply the importance sampling technique, we first recall that P_I, the probability of an event defined by the indicator function $I(\mathbf{x})$, may be written as

$$P_I = \frac{1}{N}\sum_{i=1}^{N} I(\mathbf{x}_i)L(\mathbf{x}_i) \tag{1}$$

where $L(\mathbf{x}) = p(\mathbf{x})/p^*(\mathbf{x})$ is the likelihood ratio, and $p(\mathbf{x})$ and $p^*(\mathbf{x})$ are the unbiased and biased density functions of the random vector $\mathbf{x}$. The key difficulty in applying importance sampling is to properly choose $p^*(\mathbf{x})$. The random mode coupling between the birefringent sections that comprise the optical fiber are the cause of the statistical nature of PMD. We have found that in order to bias toward large DGD, the appropriate parameters to bias are the angles θ_n between the polarization dispersion vector in the first n sections and the polarization dispersion vector in the $(n+1)$-th section at the center frequency such that $\cos\theta_n$ is biased toward one, thereby increasing the likelihood that the polarization dispersion vector at that frequency will lengthen. The angles θ_n are directly determined by the realization of the random mode coupling between the birefringent sections. Thus, the values of $\cos\theta_n$ play the role of the components of the random vector $\mathbf{x}$ in (1). The indicator function I in (1) is chosen to compute the probability of having a power penalty within any range. Thus, I is defined as one inside the desired power penalty range and zero otherwise. Specifically, we pick $\cos\theta_n$ from the following probability density function (pdf): $f(\cos\theta_n) = (\alpha/2)[(\cos\theta_n + 1)/2]^{\alpha-1}$, which corresponds to the unbiased case when $\alpha = 1$. With this pdf the likelihood ratio per section is given by $L(\cos\theta_n) = \alpha^{-1}[(\cos\theta_n + 1)/2]^{1-\alpha}$. Since the unbiased $\cos\theta_n$ are independent, the likelihood ratio of each fiber realization is equal to the product of the likelihood ratio in each section. By varying α we can obtain a significant number of samples of the random variable $\Delta\tau$ in any desired range.

We note that choosing $\alpha = 1$ corresponds to standard Monte Carlo simulations. Patching together the results at different values of α allows us to partially validate our approach as we change bias values and, in particular, to validate our approach with standard Monte Carlo simulations where they have sufficient resolution. The procedure for patching different values of α together is described in [8]–[10].

III. Numerical Results

Fig. 1 shows the pdf of the DGD of a fiber with 80 birefringent sections and 10 ps of mean DGD, $\langle\Delta\tau\rangle$. We show the DGD normalized with respect to $\langle\Delta\tau\rangle$. The unbiased probability of obtaining normalized DGD values outside the range $[0, 4]$ that we show in Fig. 1 is less than 10^{-8}. The slight deviation between the numerically calculated pdf and the Maxwellian distribution occurs because we use 80 sections rather than a much larger number [9]. We obtained this curve with only 2×10^5 samples from Monte Carlo simulations for each of the three biased distributions. The values of α are 1.0, which corresponds to standard Monte Carlo simulations, 1.4, and 1.9. The other numerical results in this letter were obtained with 10^4 samples per each value of α. In our figures, we do not include error bars because the statistical variation is too small to be visible in the log scale.

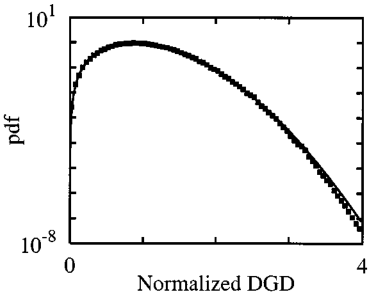

Fig. 1. The pdf of the normalized DGD plotted on a logarithmic scale. Squares are the numerical pdf and the solid line represents the Maxwellian distribution.

We now apply this method to determine the power penalty due to PMD in both a compensated and uncompensated 10-Gb/s NRZ system with 30 ps of rise time. The NRZ pulses are generated by perfect rectangular pulses filtered by a Gaussian shape filter that produces the designed rise time. The receiver is modeled by an ideal square-law photodetector and a fifth order electrical Bessel filter with 8.6-GHz bandwidth. Since our goal is to determine the operating limit of compensators with a fixed DGD element, we used the power penalty itself as the feedback parameter that we minimized. Since PMD causes pulse spreading in amplitude modulated formats, the isolated marks and spaces are the ones that suffer the highest penalty. Hence, we define the power margin as the power difference between the isolated marks and spaces. The power penalty is defined as the ratio between the back-to-back and the PMD-distorted power margins.

In Fig. 2, we show the compensated and uncompensated complement of the cumulative density function (cdfc) of the power penalty β for $\langle\Delta\tau\rangle = 30$ ps, where

$$\mathrm{cdfc}(\beta) = \int_{\beta}^{\infty} p(\beta')d\beta' \tag{2}$$

and $p(\beta)$ is the corresponding pdf. The outage probability is defined as the probability that the power penalty exceeds a certain specified margin. The value of the cdfc at a certain power penalty gives the outage probability for that amount of margin. Fig. 2(a) shows the results on a linear scale and Fig. 2(b) shows the same results on a logarithmic scale. When DGD_c, the fixed DGD element of the compensator, is set to 30 ps, which just equals the uncompensated $\langle\Delta\tau\rangle$, we observe a larger reduction of the average penalty due to PMD then when $\mathrm{DGD}_c = 75$ ps. However, we observe that the choice $\mathrm{DGD}_c = 75$ ps provides a more significant reduction of the outage probability for power penalties larger than 0.6 dB than does $\mathrm{DGD}_c = 30$ ps. The cdfc does not equal one at 0 dB because there is a finite, albeit small,

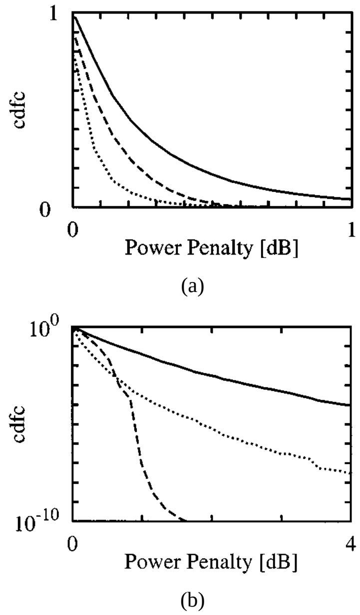

Fig. 2. Compensated and uncompensated cdfc of the power penalty due to PMD for a ensemble of fibers with $\langle \Delta\tau \rangle = 30$ ps. Solid lines are uncompensated results. Dotted lines are results for a compensator with $\mathrm{DGD}_c = \langle \Delta\tau \rangle$. Dashed lines are results for $\mathrm{DGD}_c = 2.5 \cdot \langle \Delta\tau \rangle$. (a) Results plotted on a linear scale. (b) Results plotted on a logarithmic scale.

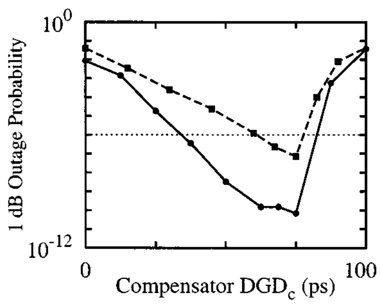

Fig. 3. The outage probability as a function of the fixed DGD element of the compensator, DGD_c. The value $\mathrm{DGD}_c = 0$ corresponds to the uncompensated case. The solid line with dots are results for $\langle \Delta\tau \rangle = 25$ ps. The dashed lines with squares are results for $\langle \Delta\tau \rangle = 30$ ps. The dotted line shows the 10^{-6} outage probability level.

probability that the PMD in the transmission line will interact with the DGD in the compensator to compress the signal.

In Fig. 3, we plot the outage probability for a 1-dB penalty as function of DGD_c for fibers with $\langle \Delta\tau \rangle = 25$ ps and $\langle \Delta\tau \rangle = 30$ ps. We see that there is an optimum value for DGD_c that minimizes the outage probability for both cases. This value is about 75 ps. The reason that the outage probability rises when DGD_c becomes larger than this optimum is that large values of DGD_c add unacceptable penalties to fiber realizations that could be adequately compensated at lower values of DGD_c. The reduction in the outage probability that the fixed DGD compensator can provide in the fiber system with $\langle \Delta\tau \rangle = 30$ ps is substantially smaller than when $\langle \Delta\tau \rangle = 25$ ps because the number of PMD realizations that the compensator cannot adequately compensate increases rapidly with the average DGD. We have also observed that is increasingly difficult to find an optimal operating point when DGD_c becomes large because the penalty depends more sensitively on the polarization controller's orientation. Thus, it is preferable to operate with the smallest possible DGD_c that produces an acceptable outage probability.

IV. Conclusion

We have studied PMD compensators with a single fixed DGD element using importance sampling. We have demonstrated that this compensator can reduce the outage probability by several orders of magnitude for NRZ signals that are transmitted in optical fibers with PMD. We have shown that the optimal value of DGD_c for realistic power penalties of 1 dB is two to three times larger than $\langle \Delta\tau \rangle$. Our results show that it is not sufficient to determine the impact of PMD compensators on the average penalty when designing realistic systems, because the average penalty is not directly related to outage probability, which is the most important design parameter. It is, therefore, crucial to accurately model the tail of the probability density function of the power penalty, and importance sampling is a technique that makes this study feasible.

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