Modeling the Saturation Induced by Broad-Band Pulses Amplified in an Erbium-Doped Fiber Amplifier

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***Abstract*—We theoretically study the saturation of a homogeneously broadened optical amplifier with a slow response time. This model approximates well the behavior of the erbium-doped fiber amplifier (EDFA). When a broad-band pulse propagates inside such amplifier the saturation is determined by the overlap between the amplifier gain profile and the pulse spectrum rather than by the energy of the pulse. This effect may significantly increase the output power of an EDFA that amplifies broad-band pulses.**

***Index Terms*—Broad-band amplifiers, nonlinear optics, optical fiber devices, optical pulses, optical saturation.**

I. Introduction

An ERBIUM-DOPED fiber amplifier (EDFA) is compact, environmentally stable, and can amplify signals in the 1.55-μm regime where most of the optical communication systems operate. The propagation of short pulses in such amplifiers and in lasers is often analyzed using the Ginzburg–Landau equation [1]–[4]. In modeling the amplifier it is often assumed that the saturation is determined by the energy of the pulses [2]–[4]. In this work we show that when a pulse with a broad spectrum propagates inside a homogeneously broadened amplifier with a slow response time, the saturation is determined by the frequency overlap between the pulse spectrum and the amplifier gain profile rather than by the pulse energy. This effect significantly increases the output power for broad-band pulses since the contribution to the gain saturation from the frequency components of the pulse that are far away from the resonant frequency is reduced. We note that the dependence of the saturation intensity on the signal frequency has been previously studied for continuous wave signals, see, e.g., [5], [6]; however, this effect has not been studied for pulses and since the saturation is a nonlinear phenomenon, pulses must be studied separately from continuous waves.

Manuscript received May 3, 1999; revised June 30, 1999. This work was supported by the Department of Energy, by the National Science Foundation, and by the Wolfson Foundation.

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Publisher Item Identifier S 1041-1135(99)07736-8.

II. Mathematical Model

The amplifier in our model is an ideal, three-level system. We assume that atoms are pumped from the lowest to the uppermost energy level and then dropped immediately to a quasistable energy level with a long lifetime on the order of a ms. Amplification is due to stimulated transitions between the intermediate and the ground levels. This system is simpler than a real EDFA but it is useful for analyzing it [3], [7], [8]. The interaction strength between the atoms and photons is modeled in this letter using the theoretical gain cross section [2]. The accuracy of the results for an EDFA might be improved by using the absorption and gain cross-sections that are measured experimentally [5], [7]. At low temperatures ($T < 77$ K) EDFA is inhomogeneously broadened due to the crystalline electric field that causes Stark-splitting in the energy levels of the Er^{3+} atoms [5], [9]. At room temperature, the large value of the homogeneous linewidth, combined with the fast relaxation time (compared to the pulse duration) among the energy levels result in an essentially uniform homogeneous saturation across the whole gain spectrum [9]. Therefore, we neglected inhomogeneous effects as is often assumed in modeling the EDFA [3], [7], [10].

The Maxwell–Bloch equations for an optical amplifier may be written [2], [3], [5]

$$\frac{\partial P}{\partial t} = -\frac{P}{T_2} - i(\omega_a - \omega_0)P - \frac{i\mu^2}{\hbar} EN \tag{1}$$

$$\frac{\partial N}{\partial t} = \frac{N_0 - N}{T_1} + \frac{1}{\hbar}\,\mathrm{Im}\{E^* P\} \tag{2}$$

where P is the slowly varying amplitude of the polarization, i.e., $p(z, t) = \frac{1}{2}\{P(z, t)\exp(-i\omega_0 t) + \text{c.c.}\}$, z is the position along the amplifier, ω_0 is the carrier frequency of the optical pulse, E is the slow varying amplitude of the electric field, μ is the dipole moment, ω_a is the resonant frequency of the atoms, N is the population inversion density, N_0 is the equilibrium value of the population inversion density, T_2 is the polarization relaxation time, and T_1 is the population relaxation time. In a three-level amplifier, the relaxation time T_1 depends on the pumping [5], so that $T_1 = \tau/(1+R\tau)$, where R is the pumping rate and τ^{-1} is the rate of the spontaneous emission.

We neglect the change in the population density N during a time scale on the order of T_2. In an EDFA, the time constant T_2 is on the order of 100 fs while the saturation energy is on the order of 100 μJ. Therefore, we will limit our analysis to pulses with a peak power of less than 10^8 W. For such pulses, (1) can be solved in the frequency domain as shown in [3]

$$P(\omega) = \epsilon_0 \chi(\omega) E(\omega) = \epsilon_0 N \frac{\sigma_s n_0 c/\omega_0}{i + T_2(\omega_0 - \omega_a - \omega)} E(\omega) \tag{3}$$

where $\sigma_s = \mu^2 \omega_0 T_2/(\varepsilon_0 n_0 \hbar c)$ is the gain cross section and n_0 is the refractive index of the amplifier.

In an EDFA, the population relaxation time is very long—on the order of a ms. Assuming that the optical pump is a continuous wave and that the pulse duration $T_0 \ll T_1$, we can neglect the effects of pumping and the spontaneous emission during the pulse duration and obtain an analytical expression for the depletion of the population density due to the pulse amplification

$$N_+ = N_- \exp\left(\frac{\epsilon_0}{\hbar} \int_{-\infty}^{+\infty} \chi_I(\omega)|E(\omega)|^2 \, d\omega\right) \tag{4}$$

where N_+ and N_- are defined as the population densities before and right after the pulse is amplified and $\chi_I \equiv 1/N \, \mathrm{Im}\{\chi(\omega)\} = -\sigma_s n_0 c/\omega_0[1 + T_2^2(\omega_0 - \omega_a - \omega)^2]$. In order to derive (4) we used the Parseval relation $\int_{-\infty}^{\infty} 1/N(t) \mathrm{Im}\{E^* P\} \, dt = \int_{-\infty}^{+\infty} \chi_I(\omega)|E(\omega)|^2 \, d\omega$. The gain coefficient g equals $-N\chi_I(\omega)\Gamma\omega_0/n_0 c$, where Γ is the confinement factor of the fiber. Therefore, the gain line shape is proportional to $\chi_I(\omega)$.

Equation (4) indicates that the saturation of the amplifier is determined by the overlap of the pulse spectrum and the gain profile that is proportional to $\chi_I(\omega)$, and not by the energy of the pulse as assumed in previous work such as in [3]. Therefore, chirp or phase distortion in the pulse will affect the saturation of the amplifier. This result can be intuitively understood by considering the frequency dependence of the interaction between photons and atoms. Different frequency components of the pulse interact differently with the atoms. The components whose frequencies are far away from the resonant frequency experience less small-signal gain; however, those components do not strongly saturate the amplifier since they interact less strongly with the atoms.

In a case that a train of similar pulses is put inside the amplifier, we can obtain an analytical expression for the steady-state population density. Assuming that $E(\omega)$ is the spectrum of a single pulse in the train, T is the repetition time of the pulse train, and N_+ and N_- are the population densities before and after the pulse arrives, defined after (4), the increase in the population density between pulses can be obtained from (2), yielding $N_- = N_0 + (N_+ - N_0)\exp(-T/T_1)$. In steady-state the depletion of the population density due to the pulse amplification is equal to the increase of the population density due to the pumping. In most EDFA applications, $T \ll T_1$ and $(\epsilon_0/\hbar)\int_{-\infty}^{\infty} \chi_I(\omega)|E(\omega)|^2 \, d\omega \ll 1$ so that the population depletion due to a single pulse is small. Using these assumptions we can simplify the expression for the steady-state population density N

$$N = \frac{N_0}{1 - (\epsilon_0 T_1/T\hbar)\displaystyle\int_{-\infty}^{\infty} \chi_I(\omega)|E(\omega)|^2 \, d\omega}. \tag{5}$$

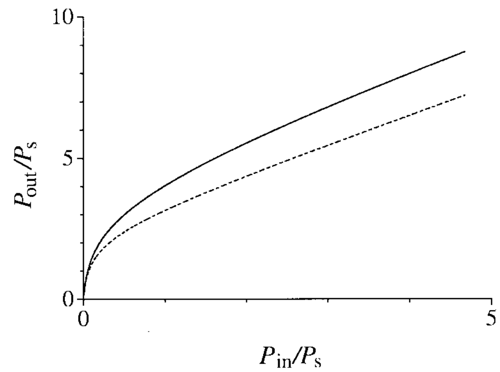

Fig. 1. Normalized output power versus normalized input power when the saturation depends on the pulse spectrum (solid line) and when the saturation depends on the pulse energy (dashed line). The bandwidth of the amplifier equals 20 nm and the small-signal gain is 25 dB. The input pulse has the spectrum of a hyperbolic-secant pulse with a FWHM of 30 nm.

Equation (5) shows again that the saturation is determined by the overlap of the pulse spectrum and the gain profile and not by the energy of the pulses.

III. Results and Discussion

In order to compare the results from the conventional saturation model and the results from the model described in this letter, we calculated the output power of the amplifier by numerically integrating the propagation equation $dE(\omega)/dz = g(\omega)E(\omega)$ where the gain coefficient g was calculated using (5). In the calculation of the saturation we took into account the frequency dependence of χ_I in our model and assumed that χ_I does not depend on the frequency for the conventional model. The full-width at half-maximum (FWHM) of the gain coefficient was 20 nm and the small-signal gain was $g_0 l = 25$ dB, where l is the amplifier length. A spontaneous emission noise factor $n_{\mathrm{sp}} = 2$ was included in our model. We assume that the carrier frequency, ω_0, equals the resonant frequency, ω_a. We neglect dispersion and nonlinear effects in the amplifier. This neglect is valid when the amplified pulse is stretched before the amplifier in order to avoid nonlinear distortion, see, e.g., [11], or when the amplified pulse has a noise-like structure with a very broad spectrum and a long duration [12], [13].

Fig. 1 shows the normalized output power as a function of the normalized input power obtained (solid line) from (5) and (dashed line) from the conventional saturation model. The normalized powers were calculated using the connection $P = 1/(P_s T)\int_{-T/2}^{+T/2} |E(t)|^2 \, dt$ where the saturation power is defined as $P_s \equiv \hbar\omega_0/\sigma_s T_1$. The pulse spectrum had a full width at half maximum of 30 nm, and the spectrum shape was equal to that of a hyperbolic-secant pulse. The results indicate that the frequency dependence of the saturation in our model significantly increases the output power. Fig. 2 shows the dependence of the output power on the full width at half maximum of the pulse spectrum. The input pulse energy was kept constant at the different pulse durations. The figure indicates that when the spectral width of the pulse is broad the output power is significantly higher than expected when

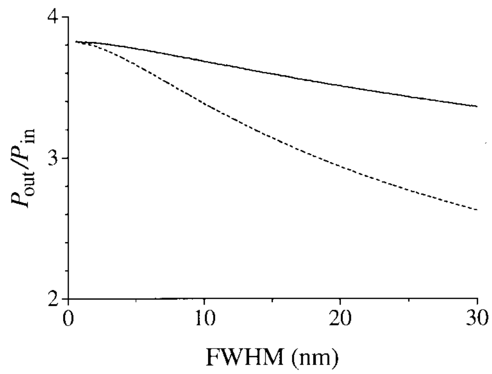

Fig. 2. Power gain versus the FWHM when the saturation depends on the pulse spectrum (solid line) and when the saturation depends on the pulse energy (dashed line). The input pulse energy does not depend on the spectral width. The bandwidth of the amplifier equals 20 nm and $P_{\rm in}/P_s = 1.4$.

only the pulse energy determines the amplifier saturation. This effect occurs because the frequency dependence of the small-signal gain decreases the output power; however the decrease is partially cancelled by the reduction of the saturation effect. The difference between the models becomes more significant when the width of the pulse spectrum is similar to the amplifier bandwidth or when the center frequency of the pulse is different from the resonant frequency.

IV. Conclusion

We have demonstrated theoretically that the saturation of homogeneously broadened amplifiers with a slow response time is determined by the overlap between the pulse spectrum and the gain profile. This effect significantly increases the output power for broad-band pulses. The increase of the power is expected to be important in generating and amplifying broad-band pulses and particularly in two-color mode-locked lasers [14] and in the noise-like mode of operation [12], [13].

Acknowledgment

The authors would like to thank Dr. G. P. Agrawal and Dr. W. L. Kath for their useful comments.

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